Circular Motion

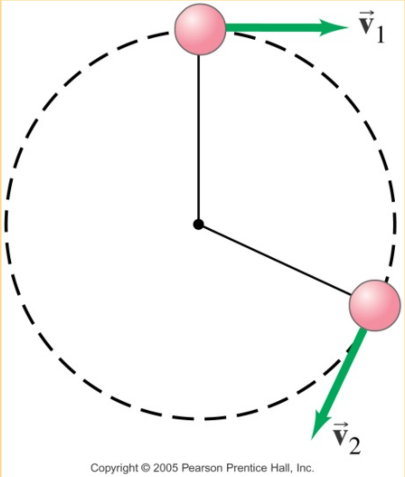
We need a net force to change the velocity
its *magnitude* or its ***direction***

Uniform Circular Motion:

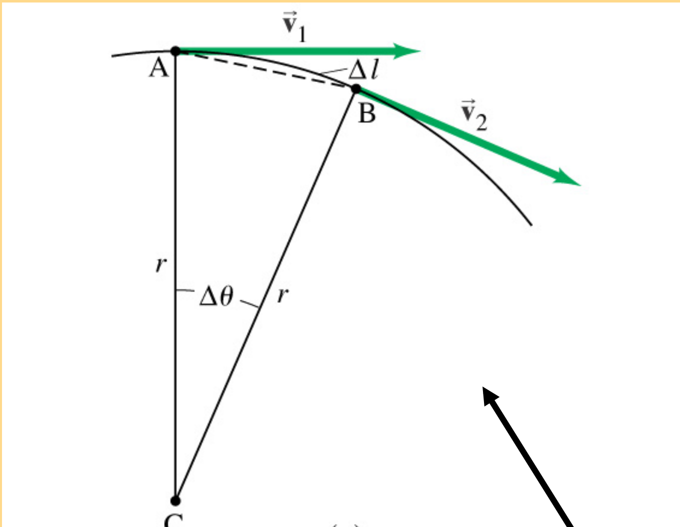
motion in a circle of constant radius at constant speed

direction is continuously changing

Instantaneous velocity is always tangent to circle.



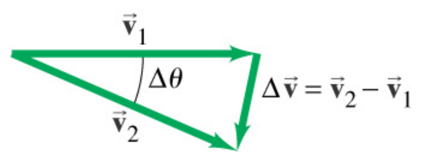
Centripetal Acceleration



(a)

$$\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{t_2 - t_1} = \frac{\Delta \vec{v}}{\Delta t}$$

Acceleration points towards the center **centripetal or radial**

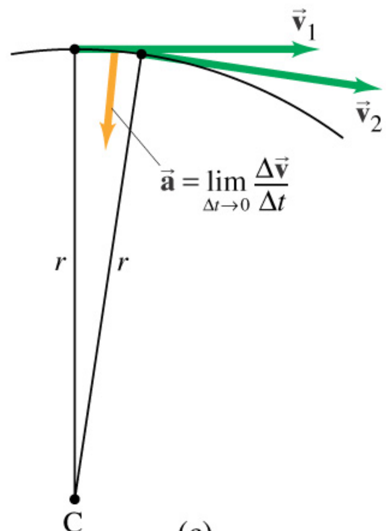


(b)

Similar triangles

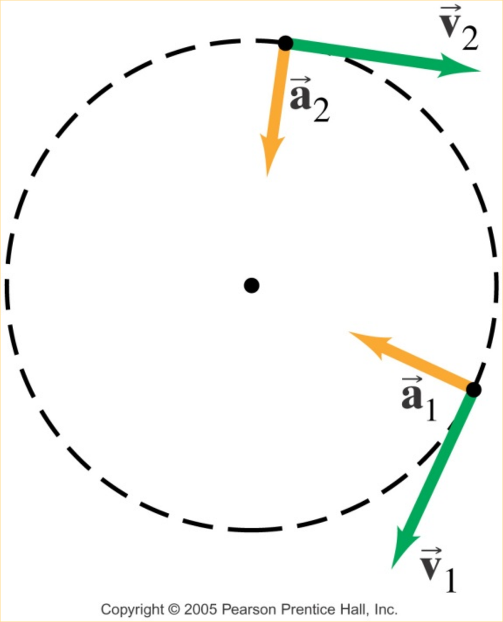
$$\frac{\Delta v}{v} = \frac{\Delta l}{r}$$

$$a_R = \frac{\Delta v}{\Delta t} = \frac{v^2}{r}$$



(c)

Period and Frequency



Period is the time to complete a revolution

Frequency is the number of revolutions per second

$$T = \frac{1}{f}$$

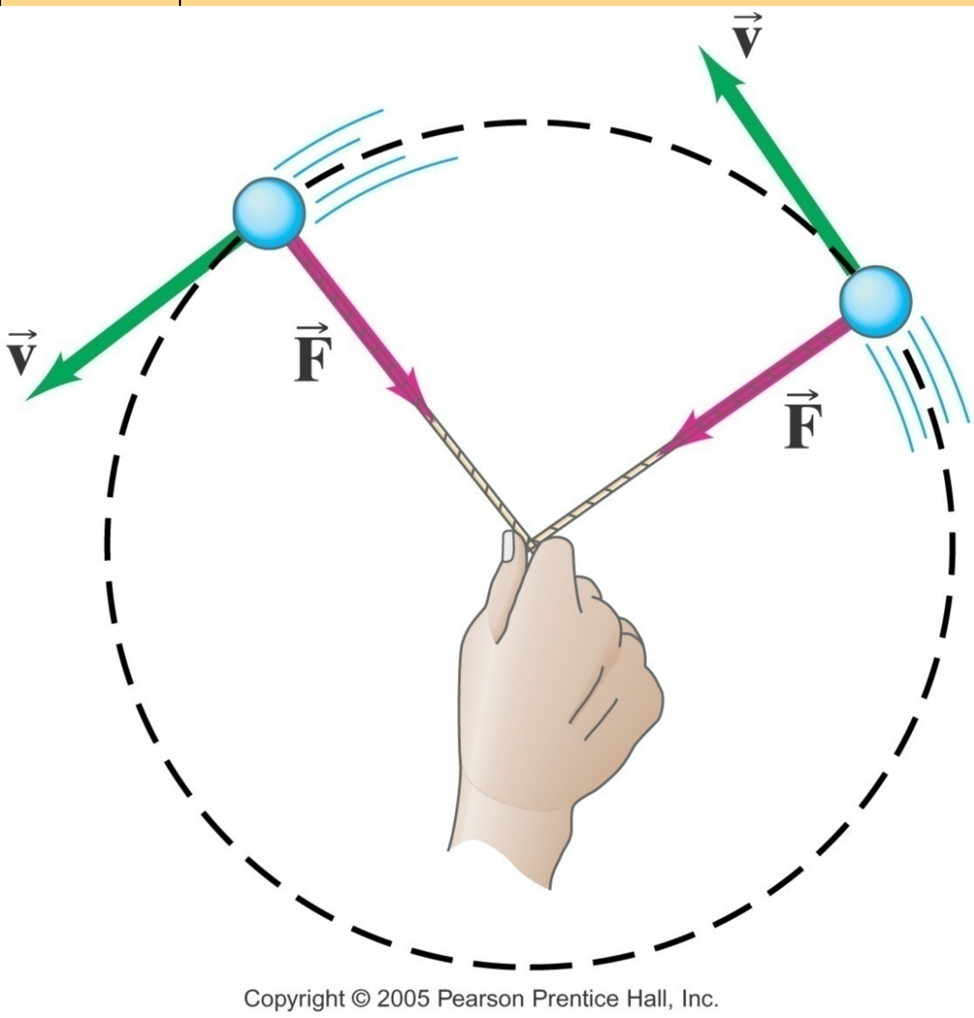
$$v = \frac{2\pi\, r}{T}$$

Ex. 5.1 A ball at the end of a string is revolving uniformly in a ***horizontal*** circle of radius 0.600 m. The ball makes 2 revolutions per second. What is its centripetal acceleration?

Ex.5.2 The Moon's circular orbit about the Earth has a radius of 384000km and a period T of 27.3 days. What is the acceleration of the Moon toward the Earth

Dynamics of Uniform Circular Motion

For an object to be in uniform circular motion, there must be a net force acting on it.



HORIZONTAL motion

We already know the acceleration, so we can immediately write the force:

$$\Sigma F_R = ma_R = m\frac{v^2}{r}$$

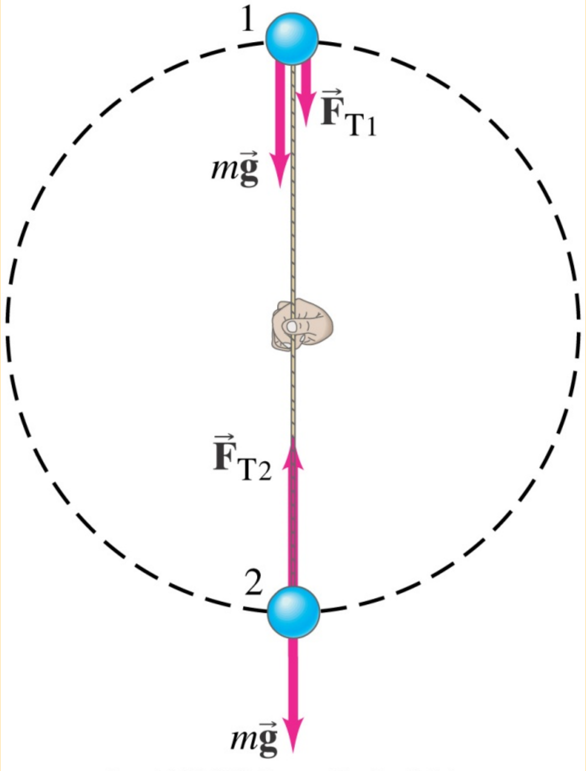
Ex.5-3 What is the force a person has to exert in Ex.5-1 if m=150g?

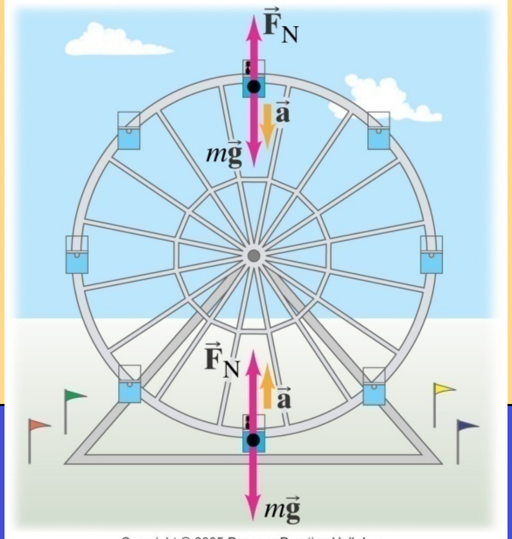
What happens if the ball is released?

It flies off tangentially

Example (Vertical Circle)

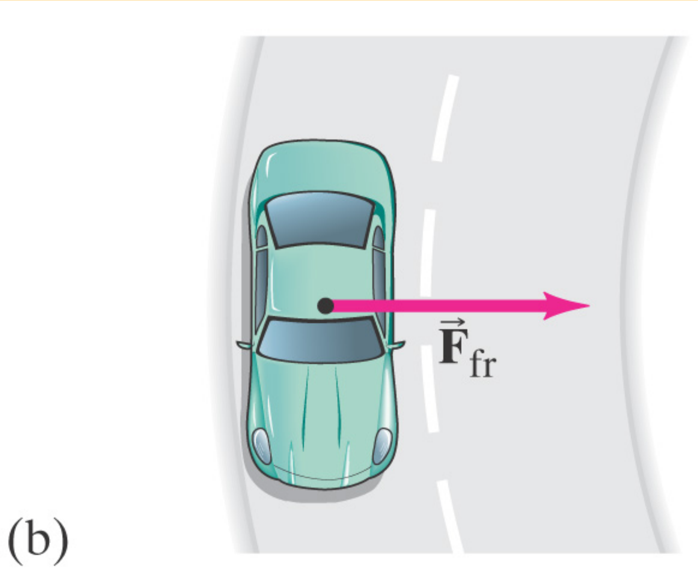
A 0.150-kg ball on the end of a 1.10 m-long cord is swung in a ***VERTICAL*** circle.
(a) Determine the minimum speed the ball must have at the top to keep the circular motion.
(b) Calculate the tension in the cord at the bottom assuming the ball is moving at twice the speed of part (a)





Ex.C (Ferris wheel)
Normal force at the top is less, more, or equal to the normal force at the bottom?

Highway Curves



It is the ***FRICTION*** force that allows a car to round a curve.
It points toward the center of the curve.

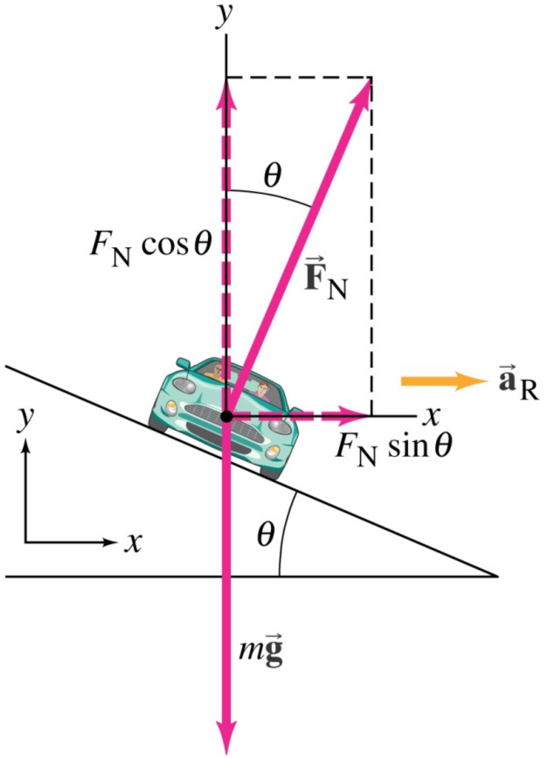
If the tires roll without sliding, the bottom of the tire is at rest against the road
Static friction force

If the static friction is not enough to keep the circular motion, the car slides.
The friction force becomes ***kinetic***

Ex. 5-6 A 1000-kg car rounds a curve on a flat road of radius 50 m at a speed of 14 m/s. Will the car follow the curve or skid? Assume:

(a) Pavement is dry, coefficient of static friction = 0.60
(b) Pavement is wet, coefficient of static friction = 0.25

Banked Curves



The banking of curves reduce the chance of skidding

For a given angle, there is one speed for which no friction is required to keep the circular motion

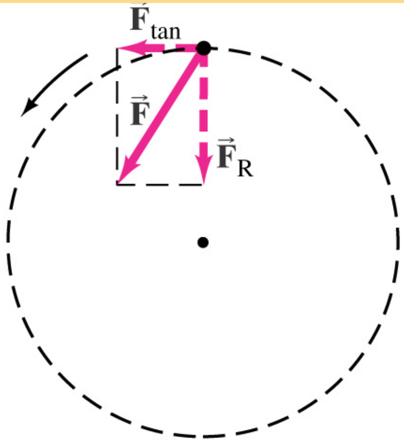
$$F_N \sin\theta = m\frac{v^2}{r}$$

Ex. 5-7 For a car traveling at speed v around a curve of radius r, determine a formula for the angle at which a road should be banked so that no friction is required

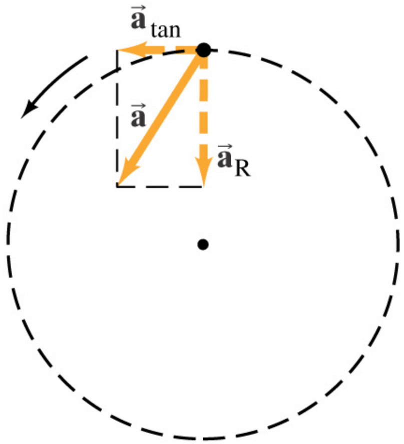
$$\tan\theta = \frac{v^2}{rg}$$

NONuniform Circular Motion

The speed of a object moving in a circle changes if the force on it has a tangential component



(a)

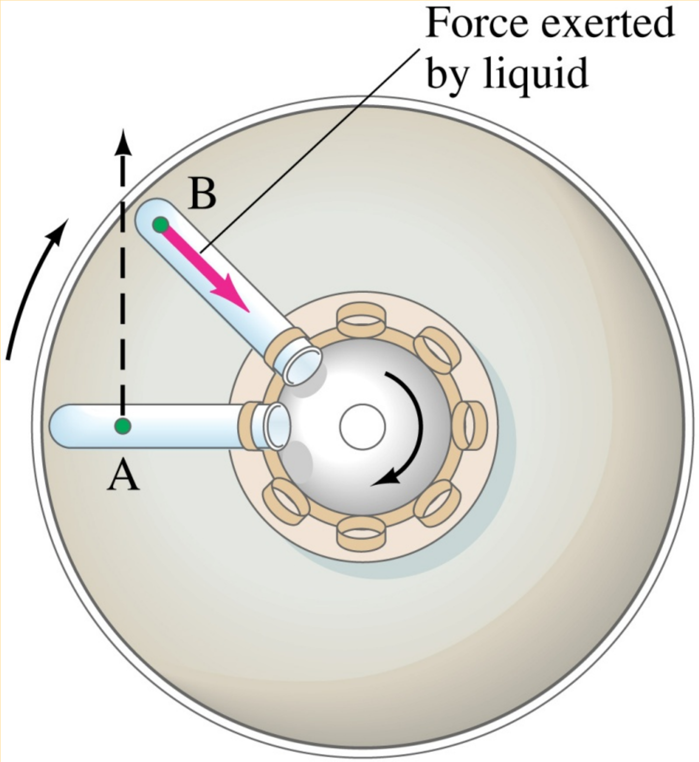


(b)

$$a_R = \frac{v^2}{r} \qquad a_{tan} = \frac{\Delta v}{\Delta t}$$

$$\boxed{a = \sqrt{a_R^{\,2} + a_{tan}^{\,2}}}$$

Centrifugation

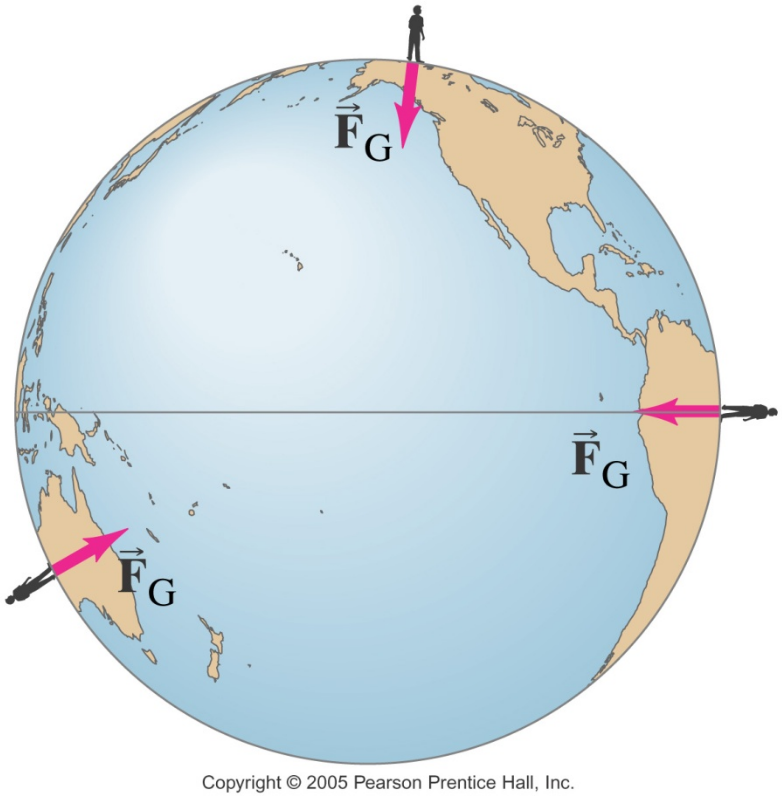


A centrifuge works by spinning very fast. This means there must be a very large centripetal force.

The resistance of the fluid does not equal the centripetal force and the particles eventually reach the bottom of the tube.

Newton's Law of Universal Gravitation

What exerts the force of gravity? Every object on Earth feels it and it always points towards the center of the Earth.



Newton's concluded that it must be the Earth that exerts the gravitational force. (legend: falling apple)

He further realized that this force must be what keeps the Moon in its orbit.

This forces decreases with the square of the distance from the Earth's center. (Ex. 5.2: a~g/3600)

Action and reaction – the force is proportional to both masses.

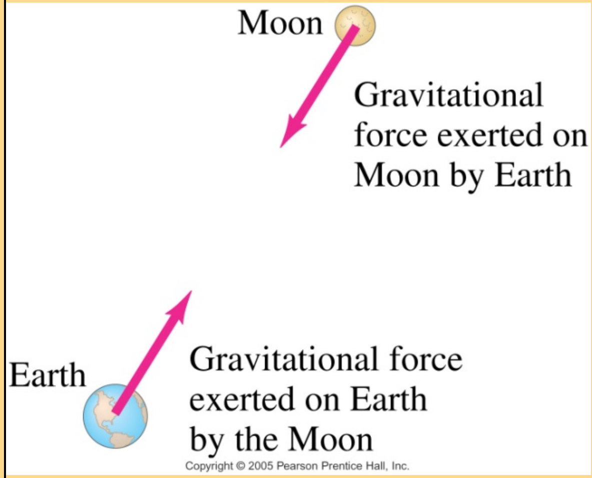
He further concluded that this force should also keep the planets in their orbits – therefore it should be a force between all objects!

Newton proposed a law of **universal** gravitation

Law of Universal Gravitation

Every particle in the universe attracts every other particle with a force that is proportional to the product of their masses and inversely proportional to the square of the distance between them. This force acts along the line joining the two particles.

Example:



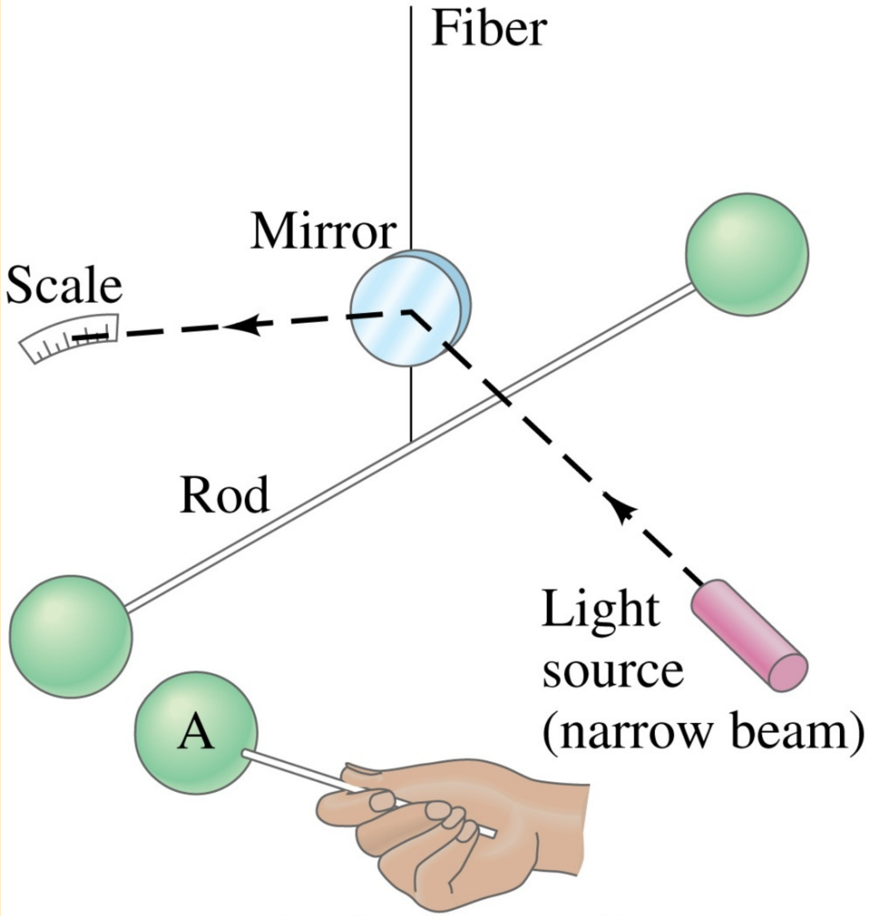
$$F = G\frac{m_1 m_2}{r^2}$$

$$G = 6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}$$

Ex. 5-10 A 50-kg person and a 75-kg person are sitting on a bench. Estimate the magnitude of the gravitational force each exerts on the other (r~0.5m).

$$F \approx 10^{-6} N$$

Cavendish Experiment



The magnitude of the gravitational constant *G* can be measured in the laboratory.

Gravity near the Earth's surface

Now we can relate the gravitational constant to the local acceleration of gravity. We know that, on the surface of the Earth:

$$mg = G\frac{mm_E}{r_E^2}$$

Solving for *g* gives:

$$\boxed{g = G\frac{m_E}{r_E^2}}$$

Now, knowing *g* and the radius of the Earth, the mass of the Earth can be calculated

$$m_E = \frac{gr_E^2}{G} = \frac{(9.80\ \mathrm{m/s^2})(6.38 \times 10^6\ \mathrm{m})^2}{6.67 \times 10^{-11}\ \mathrm{N \cdot m^2/kg^2}} = 5.98 \times 10^{24}\,\mathrm{kg}$$

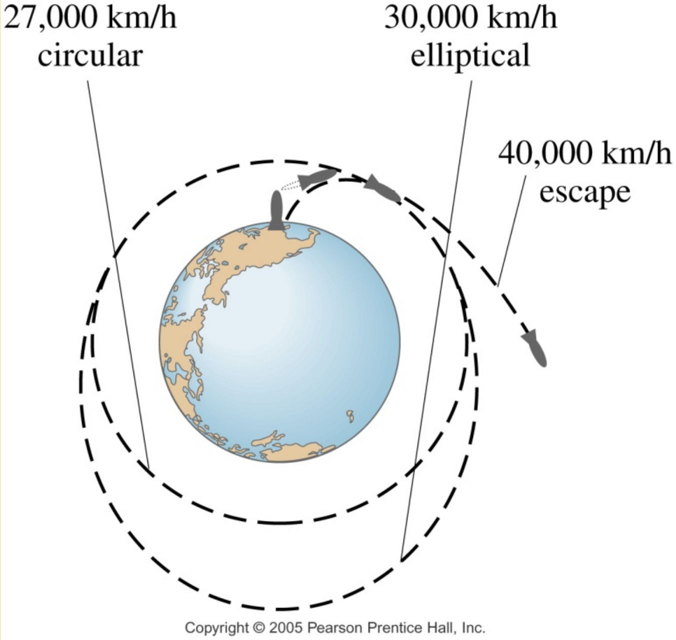
Ex. 5-13 Estimate the value of g on the top of the Mt. Everest (8850 m above Sea level), given that the radius of Earth is 6380 km.

$r = 6380\ \mathrm{km} + 8.9\ \mathrm{km} = 6.389 \times 10^6\ \mathrm{m}$ $\qquad$ $g = 9.77\,\mathrm{m/s^2}$

The value of g varies locally on the Earth's surface – this is used by geophysicists to study the structure of the Earth's crust and in mineral and oil exploration.

Satellites

Satellites are routinely put into orbit around the Earth. The tangential speed must be high enough so that the satellite does not return to Earth, but not so high that it escapes Earth's gravity altogether.



Satellite in orbit:

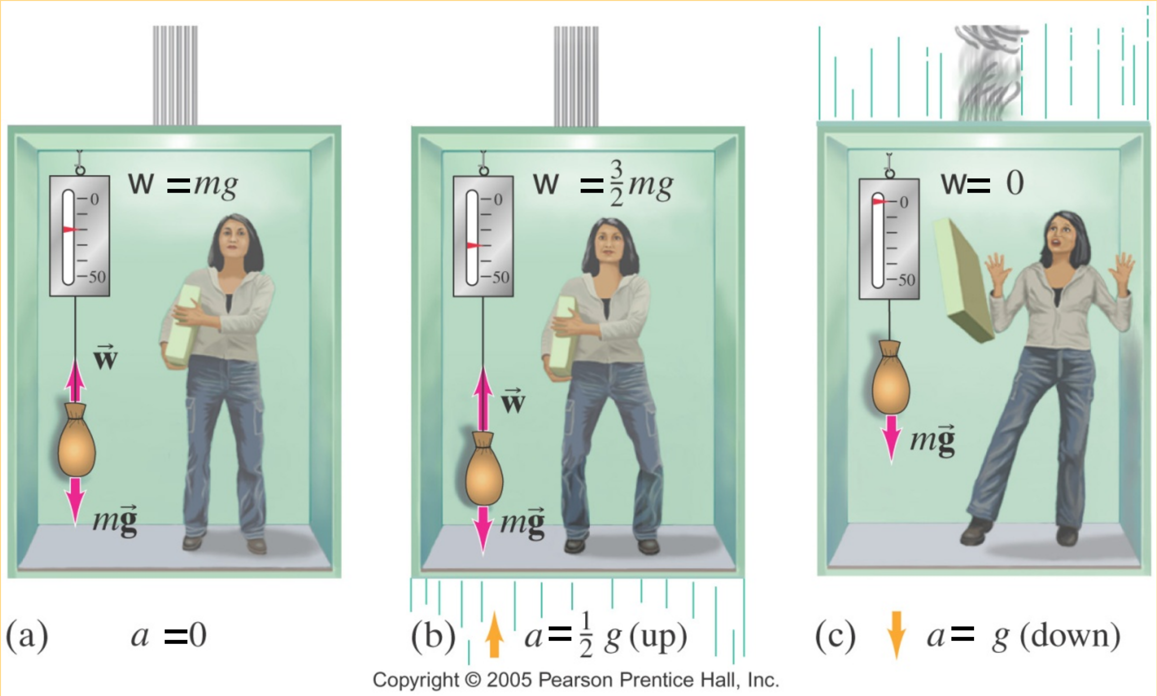
$$\boxed{G\frac{m_{sat}m_E}{r^2} = m_{sat}\frac{v^2}{r}}$$

Ex. 5-14 A geosynchronous satellite always stays above the same point on the Earth. It is used for TV, radio, weather forecasting,etc. Determine (a) the height above Earth; (b) such satellite's speed.

T=24h=86400s

r=42300km a) Above Earth:42300-6380~36000km b) v=3070m/s

“Weightlessness”



$$\sum F = ma$$
$$w - mg = ma$$
$$w = mg + ma$$

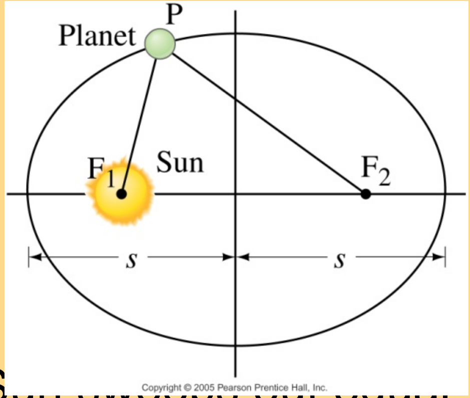
If a is + (elevator going up) the *APPARENT* weight is larger than mg

If a is – (elevator going down) the *APPARENT* weight is less than mg

If the elevator is in ***free fall***, a=-g and the scale reads zero
APPARENT WEIGHTLESSNESS: with respect to the elevator, things do not fall to the floor

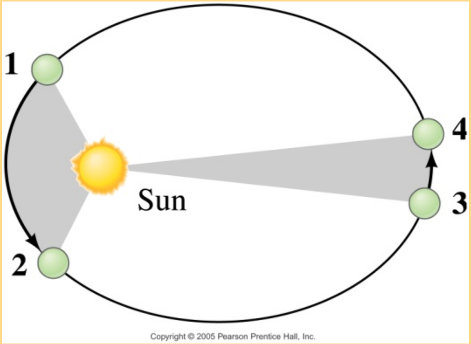
Kepler's Laws

(i) The orbit of each planet is an **ellipse**, with the **Sun at one focus.**



The Sleepwalkers – by Arthur Koestler

(ii) An imaginary line drawn from each planet to the Sun sweeps out equal areas in equal times.



(iii) $$\boxed{\left(\frac{T_1}{T_2}\right)^2 = \left(\frac{s_1}{s_2}\right)^3}$$

$$\sum F = ma$$

$$G\frac{m_1 M_S}{r_1^2} = m_1\frac{v^2}{r_1} = m_1\frac{(2\pi r_1)^2}{r_1 T_1^2}$$

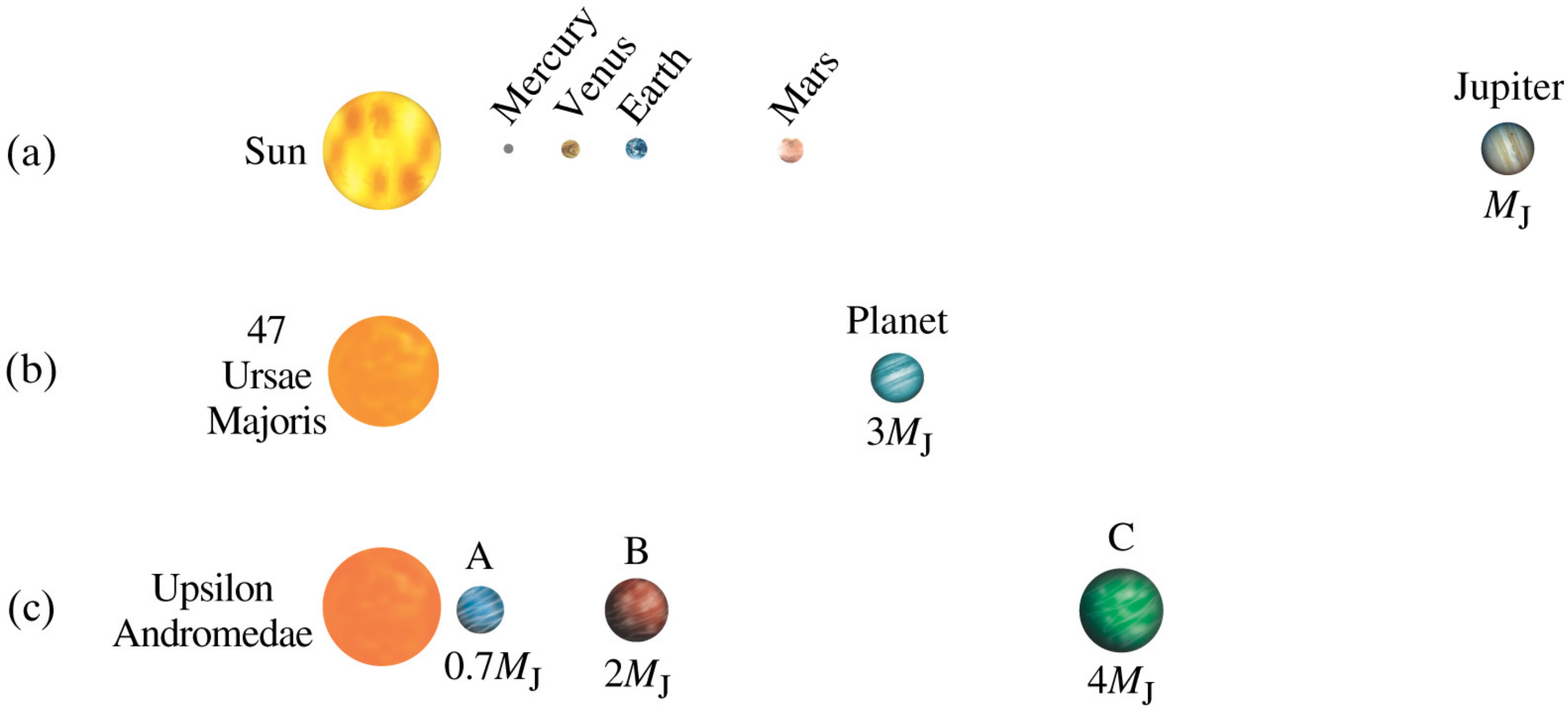
$$\frac{T_1^2}{r_1^3} = \frac{4\pi^2}{GM_S}$$

Ex. 5.15 Mars' period is 687 days=1.88 years, Earth's period is 1 year, and the distance of Earth from the Sun is 1.50x10^11 m. How far is Mars from the Sun? $r_{Ms}=1.52r_{Es}$

Ex. 5.16 Determine the mass of the Sun.

Kepler and Newton's laws

Kepler's laws can be derived from Newton's laws. Irregularities in planetary motion led to the discovery of Neptune, and irregularities in stellar motion have led to the discovery of many planets outside our Solar System.



Types of Forces in Nature

Modern physics now recognizes four fundamental forces:

1. Gravity
2. Electromagnetism
3. Weak nuclear force (responsible for some types of radioactive decay)
4. Strong nuclear force (binds protons and neutrons together in the nucleus)

More can be found in chapters 30, 31, and 32