Chapter 6 – Work and Energy

We have studied motion in terms of force, now we consider ***energy and momentum – CONSERVED*** quantities

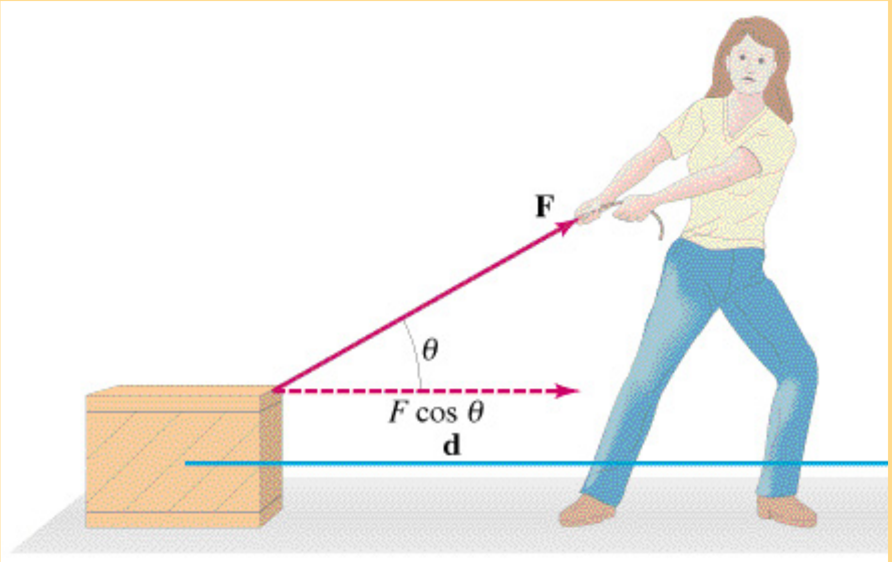
This approach is helpful when dealing with many objects and considering all forces involved become very difficult.

In this chapter we study
WORK and ENERGY
both are SCALARS

$$W = F_{\parallel} d$$

$$W = Fd\cos\theta$$

$F_{\parallel}$ component of force parallel to the displacement



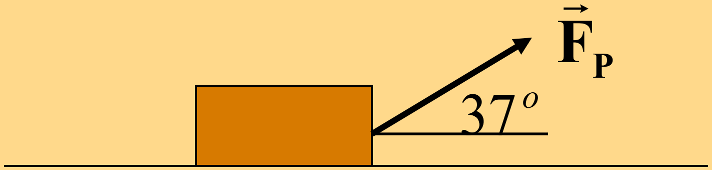
NO displacement – NO work

Units

1 N.m = 1 J (joule)

1 dyne.cm = 1 erg

Exercises



Ex. 6-1 A person pulls a 50 kg crate 40m along a horizontal floor by a constant force Fp=100N, which acts at a 37 degree angle. Ffr=50 N.

a) What is the work done by each force acting on the crate?

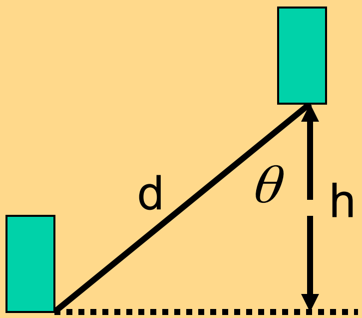
b) What is the net force done on the crate?

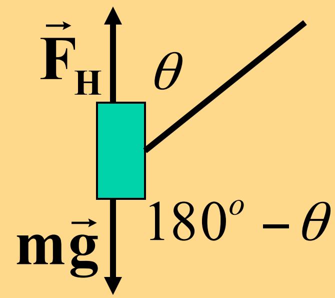
$W_G=0$, $W_N = 0$, $W_P=3200$ J, $W_{fr}=-2000$J

$W_{net}=1200$J

Ex. A A box is dragged across a floor by a force as in the figure. If the angle is increased from 0 to 90 degree, what happens to the work done to the box?

Exercises





Ex. 6-2 (a) Determine the work a hiker must do on a 15.0kg backpack to Carry it up a hill of height h=10.0m with constant speed.
Determine also (b) the work done by gravity on the backpack,
(c) the net work done on the backpack

W_H=1470J W_G=-1470J W_{net}=0

Ex.6-3 Does Earth do work on the Moon?

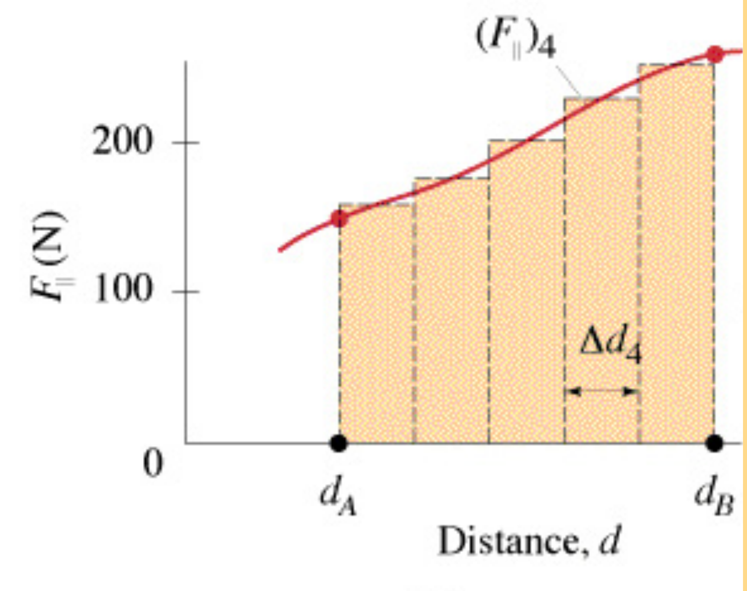
NO, because the radial force is perpendicular to tangential motion of the Moon

Work done by a Varying Force

Divide the distance into small segments and indicate for each the average $F_{||}$

$$W \approx \sum_i F_{||i} \Delta d_i$$

In the limit of very small segments, this sum becomes the **area** under the curve (integral)



Energy

There are different kinds of energy.

Kinetic energy and potential energy are examples of mechanical energy.

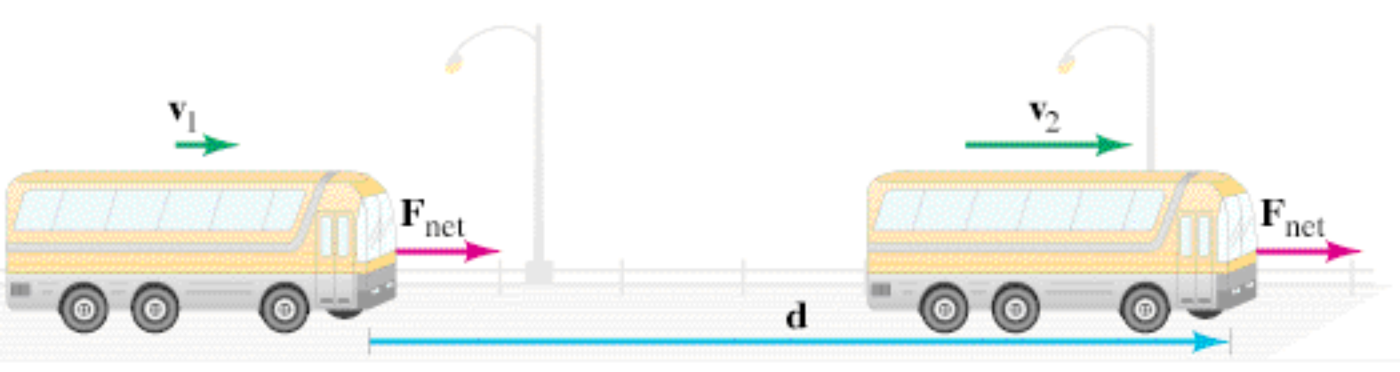
An object in motion has ***kinetic energy.***

There are also thermal energy (heat), nuclear energy, etc.

The sum of all types of energy is ***CONSERVED.***

Energy is not destroyed, only transformed.

Kinetic Energy



An object moving has kinetic energy and it can do work on another object.

$$F_{net} = ma_x; \quad a_x = \frac{v_2^2 - v_1^2}{2d} \qquad W_{net} = F_{net}d = ma_x d = \frac{mv_2^2}{2} - \frac{mv_1^2}{2}$$

$$K = \frac{mv^2}{2}$$

(translational kinetic energy)

Rotational kinetic energy – ch.8

$$W_{net} = K_2 - K_1 = \Delta K$$

Work-kinetic energy principle:

The net work done on an object is equal to the change in the object's kinetic energy

Exercises

Ex. 6-4 A 145-g baseball is thrown so that it acquires a speed of 25m/s
(a) What is its kinetic energy?
(b) What was the net work done on the ball to make it reach this speed, if it started from rest?

$K=45J$ and $W_{net}=45J$

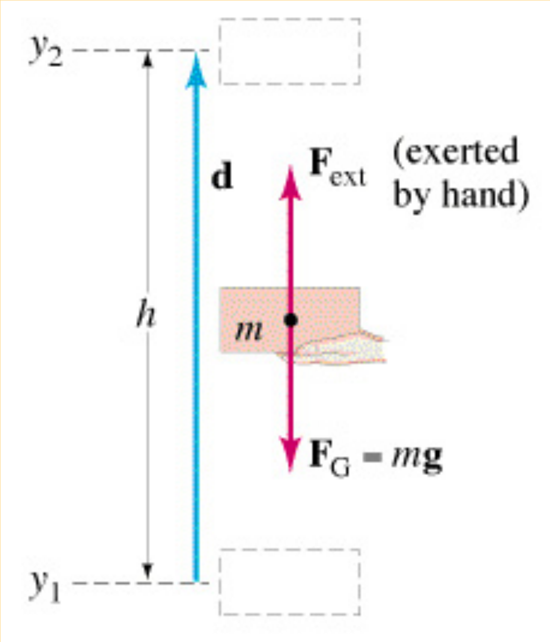
Ex. 6-5 How much work is required to accelerate a 1000-kg car from 20m/s to 30m/s

$$W_{net} = 2.5\mathbf{x}10^5 J$$

Potential Energy

Potential energy – energy associated with forces that depend on position or configuration of an object with respect to the surroundings

Examples: Gravitational potential energy (object at a certain height)
Elastic potential energy (spring)



Gravitational potential energy:

1) Hand does work on the brick
exerted force: F_{ext}

$$W_{\mathbf{ext}} = F_{\mathbf{ext}} d \cos 0 = mg(y_2 - y_1) = mgh$$

2) Gravity does work on the brick
(against the motion), F_G

$$W_{\mathbf{G}} = F_{\mathbf{G}} d \cos 180^o = -mg(y_2 - y_1) = -mgh$$

$$P_{grav} = mgy$$

gravitational potential energy

Gravitational Potential Energy

$$P_{grav} = mgy$$

The higher an object is above the ground the more gravitational potential energy it has

Work done by an external force to move the object from point 1 to 2   (a=0)

$$W_{\mathbf{ext}} = mg(y_2 - y_1) = P_2 - P_1$$

Work done by gravity as the object moves from point 1 to 2   (a=0)

$$W_{\mathbf{G}} = -mg(y_2 - y_1) = -(P_2 - P_1)$$

If the object is released, *the potential energy is transformed into kinetic energy*

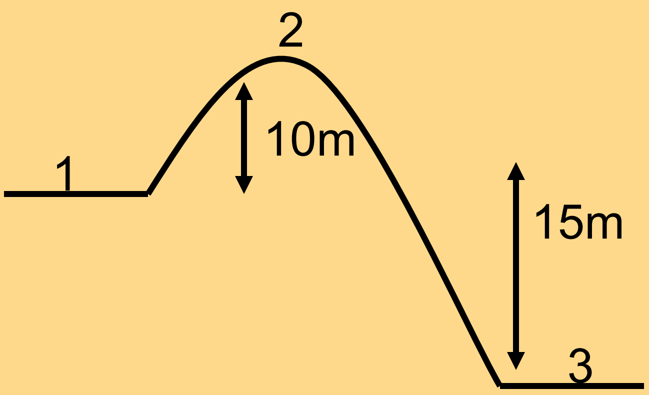
$$v^2 = 2gh$$

$$\frac{1}{2}mv^2 = mgh$$

Exercise

Reference point for zero gravitational potential energy is arbitrary

Ex.6-7 A 1000-kg roller-coaster car moves from point 1 to point 2 and then to point 3. (a) What is the gravitational potential energy at 2 and 3 RELATIVE to point 1? That is, take y=0 at point 1. (b) What is the change in potential energy when the car goes from point 2 to point 3? (c) Repeat parts (a) and (b) but take the reference point (y=0) to be at point 3.



$$P_2 = 9.8\times10^4 J \quad P_3 = -1.5\times10^5 J$$

$$P_3 - P_2 = -2.5\times10^5 J$$

$$P_2 = 2.5\times10^5 J \quad P_3 = 0$$

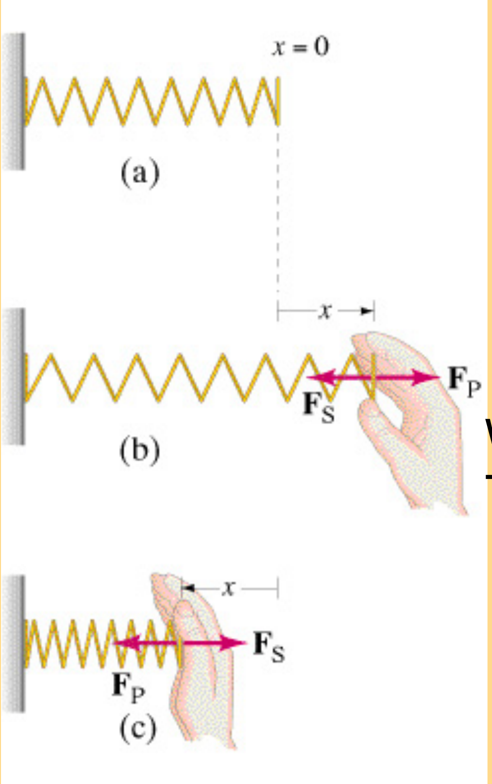
$$P_3 - P_2 = -2.5\times10^5 J$$

What is physically important is the CHANGE in potential energy, because this is what is related to work and this is what can be measured.

Elastic Potential Energy

Each form of potential energy is associated with a particular force.

The change in potential energy is the work required of an external force to move the object without acceleration between two points.



Elastic materials:

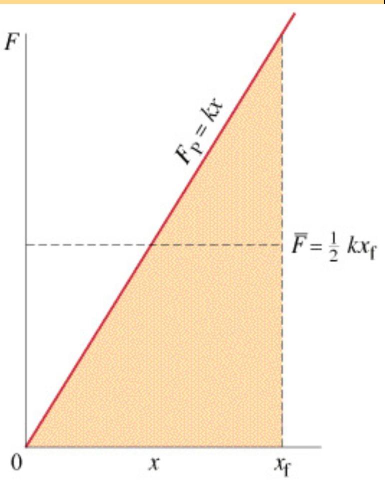
Force by the hand on the spring (k – spring stiffness constant) $F_P = kx$

Force by the spring on the hand (Hooke's law) $F_S = -kx$

Work done BY the hand to compress or stretch the spring. This force is NOT constant!

$$W = P_{el}^{f} - P_{el}^{i}$$

$$W = \tfrac{1}{2}\left(kx_f\right)x_f = \tfrac{1}{2}kx_f^{\,2}$$



Elastic potential energy $P_{el} = \tfrac{1}{2}kx^{2}$

Reference point for zero potential energy is the spring's natural position

Comments

In the examples of potential energy: object has the *POTENTIAL* to do work even though it is not actually doing it.
Energy can be *STORED* in the form of potential energy.

There is a single formula for kinetic energy, but the mathematical form for the potential energy depends on the force involved.

Conservative Forces:

forces for which the work done does *NOT* depend on the *PATH* taken, but only on the final and initial position (Ex.: gravity, elastic force).

An object that starts at a point and returns to the same point under the action of a conservative force has no net work done on it.

Nonconservative Forces:

forces for which the work done *DEPENDS* on the *PATH* taken (Ex.: friction, force exerted by a person, tension in a rope).

Work-Energy Principle

Suppose several forces, conservative and nonconservative, act on an object
W_C – work done by conservative forces
W_{NC} – work done by nonconservative forces

$$W_{\mathrm{net}} = W_{\mathrm{C}} + W_{\mathrm{NC}}$$

$$W_{\mathrm{net}} = \Delta K$$

$$W_{\mathrm{C}} + W_{\mathrm{NC}} = \Delta K$$

$$W_{\mathrm{NC}} = \Delta K - W_{\mathrm{C}}$$

Remember that the work done BY a *conservative force* (gravitational, elastic) is

$$W_{\mathrm{C}} = -\Delta P$$

$$\boxed{W_{\mathrm{NC}} = \Delta K + \Delta P}$$

Work done by nonconservative forces acting on an object is equal to the total change in kinetic and potential energies.

Conservation of Mechanical Energy

If all forces acting on an object are conservative: $W_{\mathrm{NC}} = \Delta K + \Delta P = 0$

$$\Delta K + \Delta P = 0$$

$$(K_2 - K_1) + (P_2 - P_1) = 0$$

Define a quantity E called **total MECHANICAL energy:** $E = K + P$

$$\boxed{K_2 + P_2 = K_1 + P_1}$$

$$E_2 = E_1 = Const$$

<u>Principle of conservation of mechanical energy:</u>

If only conservative forces are acting, the total mechanical energy is conserved

Problems: Cons. of Mechanical Energy

Gravitational Potential Energy

$$K_1 + P_1 = K_2 + P_2$$

$$\frac{mv_1^2}{2} + mgy_1 = \frac{mv_2^2}{2} + mgy_2$$

Ex. 6-8 A rock at 3.0 m from the ground is dropped. Calculate the rock's speed when it has fallen to 1.0 m above the ground.

v=6.3 m/s

Ex. 6-9 Assume that a roller-coaster at 40m above the ground starts from rest. Calculate (a) the speed it has at the bottom of the hill; (b) at what height it will have half this speed. Take y=0 at the bottom of the hill.

(a) V_2=28 m/s

(b) y_2=30 m

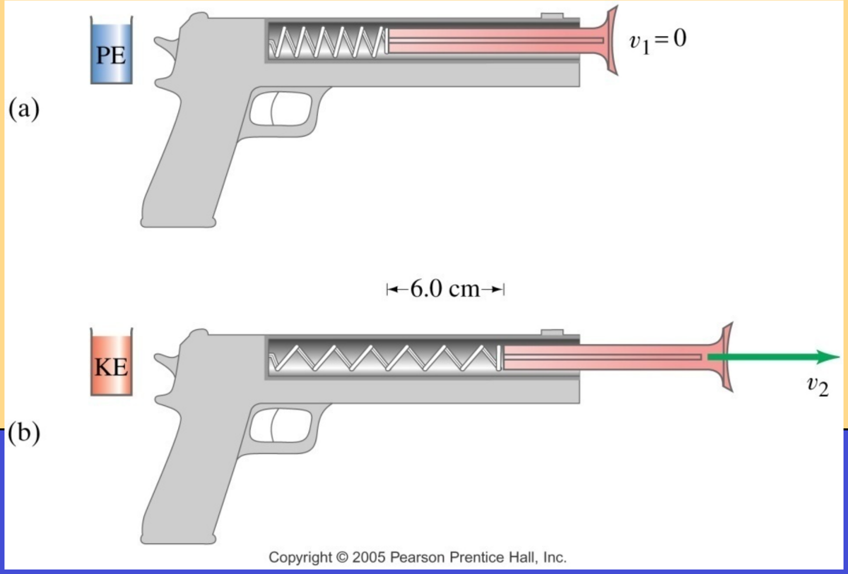
Problems: Cons. of Mechanical Energy

Elastic Potential Energy

$$K_1 + P_1 = K_2 + P_2$$

$$\frac{1}{2}mv_1^2 + \frac{1}{2}kx_1^2 = \frac{1}{2}mv_2^2 + \frac{1}{2}kx_2^2$$

Ex. 6-11 A dart of mass 0.100 kg is pressed against the spring of a toy dart gun. The spring (with spring stiffness constant k=250N/m) is compressed 6.0 cm and released. If the dart detaches from the spring when the spring reaches its natural length (x=0), what speed does the dart acquire?

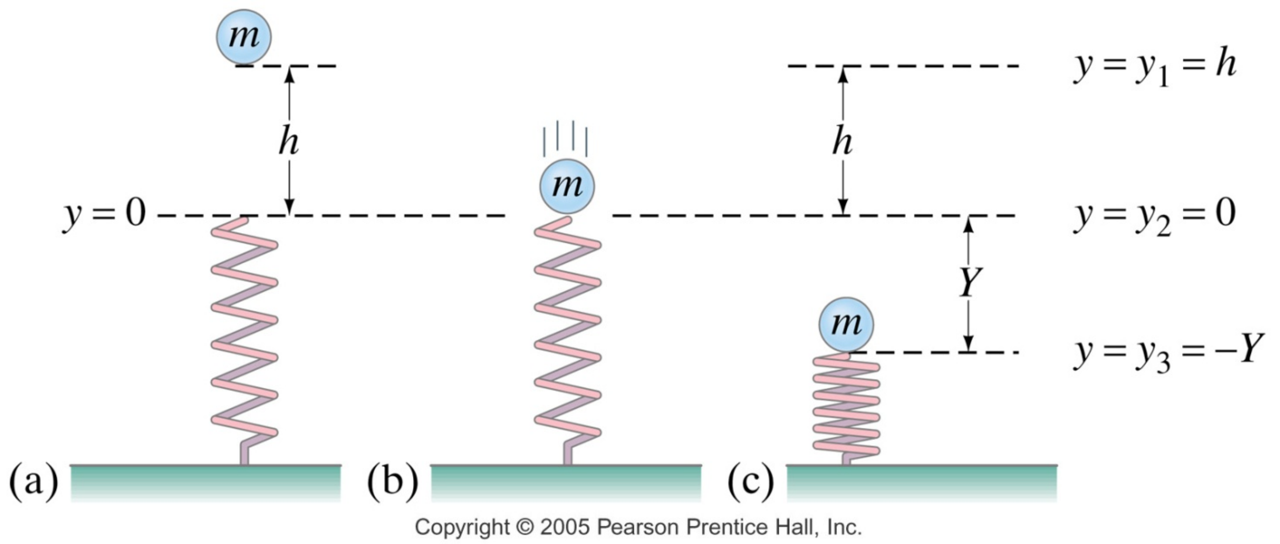


v=3.0m/s

Problems: Cons. of Mechanical Energy

Ex. 6-12 A ball of mass m=2.60 kg, starting from rest, falls a vertical distance h=55.0 cm before striking a vertical coiled spring, which it compresses an amount Y=15.0 cm. Determine the spring stiffness constant of the spring. Assume the spring has negligible mass and ignore air resistance.

$$k = \frac{2mg(h+Y)}{Y^2} = 1590 N/m$$



Conservation of Energy

Electric energy, nuclear energy, thermal energy, chemical energy.

In atomic physics, they are seen as kinetic or potential energy at the atomic level.

Thermal energy – kinetic energy of moving molecules

Energy stored in food and fuel – energy stored in the chemical bounds.

Work is done when energy is transferred from one object to another.

(spring to ball, water at the top of a damn to turbine blades, person to cart, etc)

Accounting for all forms of energy, we find that the total energy neither increases nor decreases.

Energy as a whole is conserved.

Dissipative Forces

Frictional forces reduce the total **mechanical** energy,
but NOT the total energy
They are called ***dissipative forces.***

Where do kinetic and potential energies go? **– *they become heat***

$$W_{\mathbf{NC}} = \Delta K + \Delta P = -F_{fr}d$$

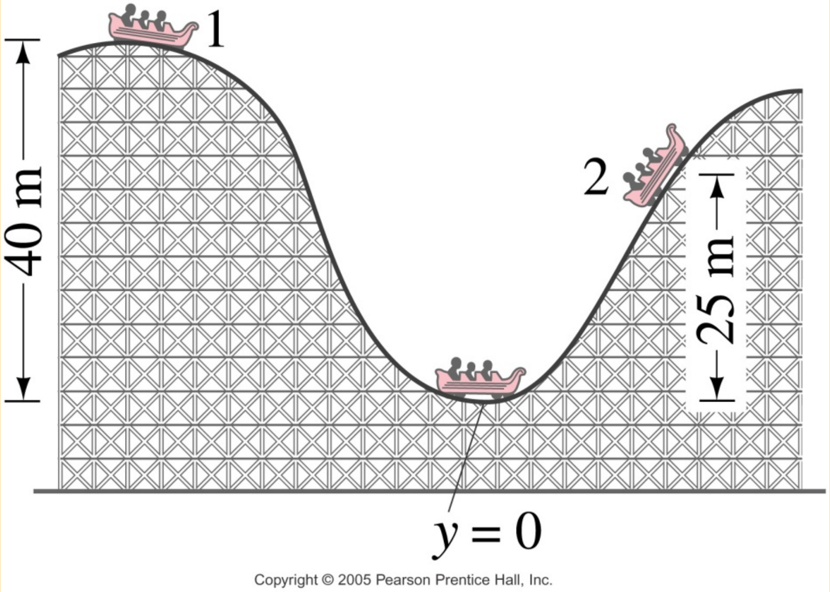
$$-F_{fr}d = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 + mgy_2 - mgy_1$$

$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 + F_{fr}d$$

Example: in the roller-coaster the initial total energy will be equal to K+P at any subsequent point along the path PLUS the thermal energy produced.

Example

Ex. 6-9 Assume that a roller-coaster of 1000 kg at 40m above the ground starts from rest. It reaches only 25m at the second hill before coming to a momentary stop. It traveled a total distance of 400 m. Estimate the average friction force.



$$\frac{1}{2}mv_1^2 + mgy_1 = \frac{1}{2}mv_2^2 + mgy_2 + F_{fr}d$$

$$F_{fr} = 370N$$

Power

Power is the rate at which work is done

$$\overline{P} = \text{average power} = \frac{\text{work}}{\text{time}} = \frac{\text{energy transformed}}{\text{time}}$$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In the SI system, the units of power are watts:

$$1\ \text{W} = 1\ \text{J/s}$$

The difference between walking and running up these stairs is power – the change in gravitational potential energy is **the same.**

Ex. 6-14 Compare the power of a 60-kg person to climb 4.5 m in 2.0s and in 4.0 s

$$\overline{P} = \frac{W}{t} = \frac{mgy}{t}$$

in 4.0s: power=660W
in 2.0s: power=1320W

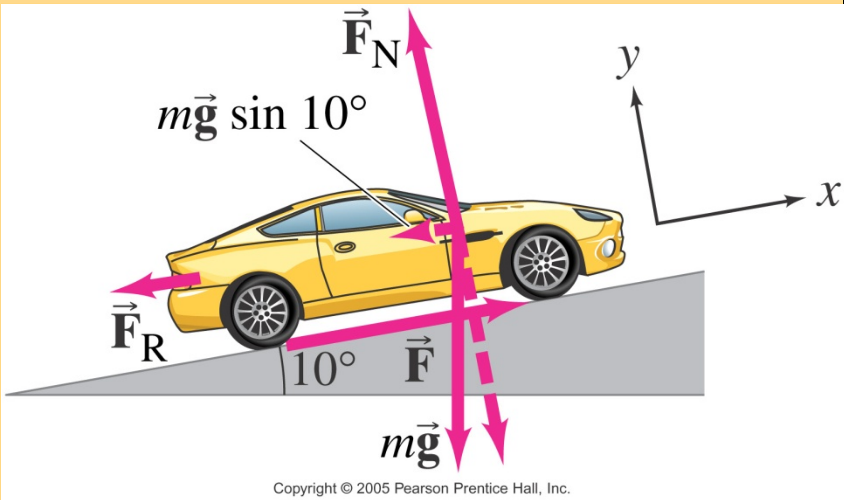
Power

Power is also needed for acceleration and for moving against the force of gravity.

The average power can be written in terms of the force and the average velocity:

$$\overline{P} = \frac{W}{t} = \frac{Fd}{t} = F\overline{v}.$$

Ex. 6-15 Calculate the power required for a 1400-kg car to climb a10 degree hill at a steady 22m/s. Assume the retarding force on the car F_R=700N.



$$F = 700N + mg\sin 10^o$$

$$\overline{P} = F\overline{v} = 6.80\times 10^4 W$$