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Manipulation of the dynamics of many-body systems via quantum control methods

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Abstract. We investigate how dynamical decoupling methods may be used to manipulate the time evolution of quantum many-body systems. These methods consist of sequences of external control operations designed to induce a desired dynamics. The systems considered for the analysis are one-dimensional spin-1/2 models, which, according to the parameters of the Hamiltonian, may be in the integrable or nonintegrable limits, and in the gapped or gapless phases. We show that an appropriate control sequence may lead a chaotic chain to evolve as an integrable chain and a system in the gapless phase to behave as a system in the gapped phase. A key ingredient for the control schemes developed here is the possibility to use, in the same sequence, different time intervals between control operations.

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Contents

1. Introduction

The study of one-dimensional quantum many-body systems out of equilibrium offers several fundamental challenges. A main question is whether an isolated quantum system can or cannot thermalize after a quench. While a relationship between quantum chaos and thermalization is certain to exist [1]–[6], an ongoing controversy is the occurrence of thermalization for a chaotic system in the gapped phase [7]–[9]. In terms of transport properties, the discussions are associated with the expected dynamical behavior of systems in the integrable and nonintegrable regimes [10]–[20], and in the gapped and gapless phases [21, 22]. Extraordinary technological advances have motivated the use of experiments to address these issues. In this work, we show how these experiments may benefit from dynamical decoupling methods to accurately reach the different regimes and phases of interest.

The fundamental questions raised above have found a test bed in experiments with ultracold quantum gases in optical lattices [23] and with magnetic compounds [24]–[26]. Both systems require a high level of controllability. In optical lattices, control of atomic transport is achieved with time-dependent variations of the depth and phase of the lattice potential, whereas in magnetic systems the preparation of samples where impurities and unwanted couplings are negligible is key for obtaining a desired transport behavior. In addition to monitoring experimental conditions, we advocate the use of sequences of external electromagnetic field pulses to manipulate the dynamics of the systems. This method of quantum control has long been employed in nuclear magnetic resonance (NMR) spectroscopy [27, 28]. There, the Hamiltonian of nuclear spin systems is modified with sequences of unitary transformations (radio-frequency pulses), which constantly rotate the spins and remove or rescale selected terms of the original Hamiltonian. These techniques have acquired a new dimension in the context of quantum information processing [29, 30], where the studies of transport of information [31] and the manipulation of electron spins with ultrafast laser pulses [32, 33] are among the latest developments. Recently, these methods have also been proposed for the control of atomic transport in spinor optical lattices [34].

In the present work, we use the theory developed in NMR to construct sequences of control operations that appropriately modify the Hamiltonian of a one-dimensional quantum many-body

system and force it to follow a desired dynamics. The systems analyzed are modeled by spin-1/2 chains, which may also be mapped onto systems of spinless fermions or hardcore bosons. We focus on limits that are of interest for studies of transport behavior and thermalization. Specifically, our goals are (i) to dynamically remove the effects of terms leading to the onset of chaos, such as disorder and frustration, and compel the chain to evolve as an integrable system, as introduced in [18, 19], and (ii) to dynamically increase the effects of terms that can drive the system across a transition from the gapless to the gapped phase (and vice versa), aiming, for example, at forcing a conductor to behave as an insulator or at bringing a chaotic system into the gapped phase. For the gapless–gapped transition, the strengths of the coupling terms in the Hamiltonian are modified by applying to the target system sequences with different time intervals between the control operations. The idea of varying the time separation between pulses has been exploited in sequences created to suppress the couplings between a qubit and its environment [35, 36]. Contrary to that, the systems considered here are isolated from the environment, and our objective is not necessarily to completely eliminate couplings, but instead to control their strengths.

This paper is organized as follows. Section 2 describes the model under investigation. Section 3 reviews a technique to design control sequences, which is based on the average Hamiltonian theory. Section 4 presents in detail sequences that dynamically lead the system studied across a transition between the nonintegrable and integrable limits and between the gapless and gapped phases. Numerical results are shown for the time evolution of local magnetization based on a particular initial state. A discussion about control metrics and the dependence of the outcomes on initial states is left for the appendix. Conclusions are given in section 5.

2. Description of the model

We study a one-dimensional spin-1/2 system with open boundary conditions described by the Hamiltonian

$$H_0 = H_z + H_{\mathrm{NN}} + \alpha H_{\mathrm{NNN}}, \tag{1}$$

where

$$H_z = \sum_{n=1}^{L} \epsilon_n S_n^z,$$

$$H_{\mathrm{NN}} = \sum_{n=1}^{L-1} J\left[\left(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y\right) + \Delta S_n^z S_{n+1}^z\right],$$

$$H_{\mathrm{NNN}} = \sum_{n=1}^{L-2} J\left[\left(S_n^x S_{n+2}^x + S_n^y S_{n+2}^y\right) + \Delta S_n^z S_{n+2}^z\right].$$

Above, $\hbar$ is set equal to 1, L is the number of sites and $S_n^{x,y,z} = \sigma_n^{x,y,z}/2$ are the spin operators at site n, $\sigma_n^{x,y,z}$ being the Pauli matrices.

(i) The parameter ϵ_n corresponds to the Zeeman splitting of spin n, as determined by a static magnetic field in the z-direction. The system is clean when all ϵ_ns are equal ($\epsilon_n = \epsilon$) and it is disordered when at least one spin n has an energy splitting different from the others ($\epsilon_n \neq \epsilon$). We therefore consider only on-site disorder.

Table 1. Parameters of the spin-1/2 chain described by equation (1).

ϵ_n	Energy splitting of spin n
J	Coupling strength
Δ	Ratio between Ising interaction and flip-flop term
α	Ratio between NNN and NN couplings

(ii) H_{NN}, also known as the XXZ Hamiltonian, corresponds to the nearest-neighbor (NN) exchange; $S_n^z S_{n+1}^z$ is the Ising interaction and $S_n^x S_{n+1}^x + S_n^y S_{n+1}^y$ is the flip-flop term. The coupling strength J and the anisotropy Δ are assumed positive, thus favoring antiferromagnetic order. By varying Δ in $H_0 = H_{\mathrm{NN}}$ we may go from the gapless phase ($\Delta < 1$) to the gapped phase ($\Delta > 1$), which, at $T = 0$, is followed by the metal–insulator Mott transition [37]. In the particular case of $\Delta = 0$, we deal with the XY model, whereas $\Delta = 1$ leads to the isotropic Heisenberg Hamiltonian [38].

(iii) The parameter α refers to the ratio between the next-nearest-neighbor (NNN) exchange, as determined by H_{NNN}, and the NN couplings. The inclusion of NNN antiferromagnetic exchange frustrates the chain, since NN exchange favors ferromagnetic alignment between the second neighbors. Different phenomena are expected in the presence of frustration [39]. In the case of a closed isotropic system, for instance, a critical value $\alpha_{\mathrm{c}} \approx 0.241$ exists, which separates the spin fluid phase ($\alpha < \alpha_{\mathrm{c}}$) from the dimer phase ($\alpha > \alpha_{\mathrm{c}}$) [40, 41].

A clean one-dimensional spin-1/2 model with NN exchange only is integrable and solved by Bethe Ansatz method [42]. The addition of defects [43], even if just one in the middle of the chain [44], or the inclusion of NNN couplings [45, 46] may lead to the onset of quantum chaos.

3. Design of dynamical decoupling sequences

Dynamical decoupling methods consist in the application of sequences of unitary transformations to generate a desired effective propagator at time t. The first experimental implementations of sequences of electromagnetic pulses aiming at controlling the dynamics of molecular systems appeared in the context of NMR spectroscopy and correspond to the so-called spin-echo and Carr–Purcell sequences [47, 48]. The basic idea of the method is to add a time-dependent control Hamiltonian, $H_{\mathrm{c}}(t)$, to the original (time-independent, in our case) Hamiltonian of the system, H_0, so that the system evolution is dictated by the following propagator:

$$U(t) = \mathcal{T} \exp\left\{-\mathrm{i}\int_0^t [H_0 + H_{\mathrm{c}}(u)]\,\mathrm{d}u\right\},$$

where $\mathcal{T}$ stands for time ordering. This operator may also be written as [27, 28]

$$U(t) = U_{\mathrm{c}}(t)\mathcal{T} \exp\left\{-\mathrm{i}\int_0^t [U_{\mathrm{c}}^{-1}(u) H_0 U_{\mathrm{c}}(u)]\,\mathrm{d}u\right\},$$

where

$$U_{\mathrm{c}}(t) = \mathcal{T} \exp\left[-\mathrm{i}\int_0^t H_{\mathrm{c}}(u)\,\mathrm{d}u\right]$$

depends only on the introduced perturbation $H_c(t)$ and is known as the control propagator. We deal with cyclic control sequences with cycle time T_c. This means that the perturbation and the control propagator are periodic, that is, $H_c(t+nT_c) = H_c(t)$ and $U_c(t+nT_c) = U_c(t)$, $n \in \mathbb{N}$. Since $U_c(0) = \mathbb{1}$, it follows that the control propagator at $T_n = nT_c$ is $U_c(nT_c) = \mathbb{1}$. As a result, the general propagator at T_c is simply $U(T_c) = \mathcal{T}e^{-i\int_0^{T_c}[U_c^{-1}(u)H_0U_c(u)]\,du}$ and for n cycles

$$U(nT_c) = U(T_c)^n.$$

Therefore, the evolution of the system at any time $T_n = nT_c$ requires only the knowledge of the propagator after one cycle [27, 28].

We consider the ideal scenario of arbitrarily strong and instantaneous control operations, known in NMR as hard pulses and in the field of quantum information as bang-bang control (the latter term was borrowed from classical control theory) [29]. Each pulse P_{k+1} is applied after a time interval of free evolution $\tau_{k+1} = t_{k+1} - t_k$, where $k \in \mathbb{N}$ and $t_0 = 0$. For a cyclic sequence with m pulses, the evolution operator at $T_c = m\tau$ is then given by

$$\begin{aligned}
U(T_c) &= P_m U(t_m, t_{m-1}) P_{m-1} U(t_{m-1}, t_{m-2}) \ldots P_2 U(t_2, t_1) P_1 U(t_1, 0) \\
&= \underbrace{(P_m P_{m-1} \ldots P_1)}_{U_c(T_c) = \mathbb{1}} \underbrace{(P_{m-1} \ldots P_1)^\dagger U(t_m, t_{m-1})(P_{m-1} \ldots P_1)}_{e^{-i(P_{m-1}\ldots P_1)^\dagger H_0 (P_{m-1}\ldots P_1)\tau_m}} \ldots \\
&\quad \underbrace{(P_2 P_1)^\dagger U(t_3, t_2)(P_2 P_1)}_{e^{-i(P_2P_1)^\dagger H_0(P_2P_1)\tau_3}} \underbrace{P_1^\dagger U(t_2, t_1) P_1}_{e^{-iP_1^\dagger H_0 P_1 \tau_2}} \underbrace{U(t_1, 0)}_{e^{-iH_0\tau_1}} \\
&= \exp\left[-iH_{m-1}\tau_m\right] \ldots \exp\left[-iH_2\tau_3\right] \exp\left[-iH_1\tau_2\right] \exp\left[-iH_0\tau_1\right] \\
&= \exp\left[-i\bar{H}T_c\right]. \qquad (2)
\end{aligned}$$

Above, the notation $H_{m-1} = (P_{m-1} \ldots P_1)^\dagger H_0 (P_{m-1} \ldots P_1)$ has been used. The Hamiltonian $\bar{H}$ in the last line is obtained via Baker–Campbell–Hausdorff expansion and is referred to as the average Hamiltonian [27, 28].

The lowest order term of $\bar{H}$ in τ corresponds to

$$\bar{H}^{(0)} = \sum_{k=0}^{m-1} \frac{\tau_{k+1}}{T_c} H_k. \qquad (3)$$

In designing a dynamical decoupling sequence, the primary goal is to guarantee the proximity of $\bar{H}^{(0)}$ to the desired Hamiltonian. This is the focus of the current work. We note, however, that various strategies exist to eliminate some of the higher order terms [27, 28] or to reduce the accumulation of errors arising from imperfect averaging [49]–[52].

4. Modifying the system evolution

In this section, we explore how dynamical decoupling schemes may induce transitions between different regimes and phases of a quantum many-body system. We aim at dynamically reshaping the original Hamiltonian (1) to make it as close as possible to a desired Hamiltonian H_w (the subscript 'w' is used whenever we refer to parameters and quantities associated with the wanted evolution). Our numerical results are performed in the full Hilbert space of dimension $D = 2^L$. We cannot take advantage of the symmetries of H_0 (1), such as conservation of total spin in

the z-direction, to reduce this dimension, because the intervals of free evolution of the pulsed system involve Hamiltonians [H_k in equation (3)] that are different from the original H_0.

4.1. Nonintegrable versus integrable systems

To start, we discuss sequences of control pulses that average out the terms in Hamiltonian (1) that may lead to the onset of chaos, namely on-site disorder and NNN exchange.

4.1.1. Annihilating the effects of on-site disorder: $H_z + H_{\rm NN} \to H_{\rm NN}$. The first and simplest sequence that we present was proposed in [18, 19] and it aims at eliminating the effects of on-site disorder. Consider a disordered system described by $H_z + H_{\rm NN}$ with one or more sites having $\epsilon_n \neq \epsilon$. In order to recover the transport behavior of a clean chain given by $H_{\rm NN}$, we apply, after every τ, control operations P, which rotate all spins by 180° (π-pulses) around a direction perpendicular to z. Around x, for instance, the pulses are

$$P_x = \exp\left(-\mathrm{i}\pi \sum_{n=1}^{L} S_n^x\right) = \exp(-\mathrm{i}\pi S^x). \tag{4}$$

The propagator at cycle time $T_{\rm c} = 2\tau$ becomes

$$\begin{aligned} U(T_{\rm c}) &= P_x \exp\left[-\mathrm{i}\,(H_z + H_{\rm NN})\,\tau\right] P_x \exp\left[-\mathrm{i}\,(H_z + H_{\rm NN})\,\tau\right] \\ &= (-1)\exp\left[-\mathrm{i}\,(-H_z + H_{\rm NN})\,\tau\right]\exp\left[-\mathrm{i}\,(H_z + H_{\rm NN})\,\tau\right], \end{aligned}$$

which leads to the following dominant term of the average Hamiltonian,

$$\bar{H}^{(0)} = H_{\rm NN},$$

as desired.

4.1.2. Annihilating the effects of NNN exchange: $H_{\rm NN} + H_{\rm NNN} \to H_{\rm NN}$. A sequence to eliminate the effects of NNN exchange (and also on-site disorder) was introduced in [18] and is further investigated here. It has eight π-pulses equally separated ($T_{\rm c} = 8\tau$) and applied to selected spins, which therefore assumes site addressability. The pulses are

$$\begin{aligned} P_1 = P_3 &= \prod_{k=0}^{\lfloor (L-1)/4 \rfloor} \mathrm{e}^{-\mathrm{i}\pi S^x_{1+4k}} \prod_{k=0}^{\lfloor (L-2)/4 \rfloor} \mathrm{e}^{-\mathrm{i}\pi S^x_{2+4k}}, \\ P_2 = P_4 &= \prod_{k=0}^{\lfloor (L-3)/4 \rfloor} \mathrm{e}^{-\mathrm{i}\pi S^y_{3+4k}} \prod_{k=0}^{\lfloor (L-4)/4 \rfloor} \mathrm{e}^{-\mathrm{i}\pi S^y_{4+4k}}, \\ P_5 = P_7 &= \prod_{k=0}^{\lfloor (L-2)/4 \rfloor} \mathrm{e}^{-\mathrm{i}\pi S^x_{2+4k}} \prod_{k=0}^{\lfloor (L-3)/4 \rfloor} \mathrm{e}^{-\mathrm{i}\pi S^x_{3+4k}}, \\ P_6 = P_8 &= \prod_{k=0}^{\lfloor (L-1)/4 \rfloor} \mathrm{e}^{-\mathrm{i}\pi S^y_{1+4k}} \prod_{k=0}^{\lfloor (L-4)/4 \rfloor} \mathrm{e}^{-\mathrm{i}\pi S^y_{4+4k}}. \end{aligned} \tag{5}$$

They change the signs of the coupling terms in each interval of free evolution according to table 2.

Table 2. Sign changes of the coupling terms in $H_{\rm NN}$ and $H_{\rm NNN}$ according to pulses (5).

	$(0,\tau)$	$(\tau,2\tau)$	$(2\tau,3\tau)$	$(3\tau,4\tau)$	$(4\tau,5\tau)$	$(5\tau,6\tau)$	$(6\tau,7\tau)$	$(7\tau,8\tau)$
NN couplings								
$S^x_{2k+1}S^x_{2k+2}$	+	+	+	+	+	+	−	−
$S^x_{2k+2}S^x_{2k+3}$	+	+	−	−	+	+	+	+
$S^y_{2k+1}S^y_{2k+2}$	+	+	+	+	+	−	−	+
$S^y_{2k+2}S^y_{2k+3}$	+	−	−	+	+	+	+	+
$S^z_{2k+1}S^z_{2k+2}$	+	+	+	+	+	−	+	−
$S^z_{2k+2}S^z_{2k+3}$	+	−	+	−	+	+	+	+
NNN couplings								
$S^x_{k+1}S^x_{k+3}$	+	+	−	−	+	+	−	−
$S^y_{k+1}S^y_{k+3}$	+	−	−	+	+	−	−	+
$S^z_{k+1}S^z_{k+3}$	+	−	+	−	+	−	+	−

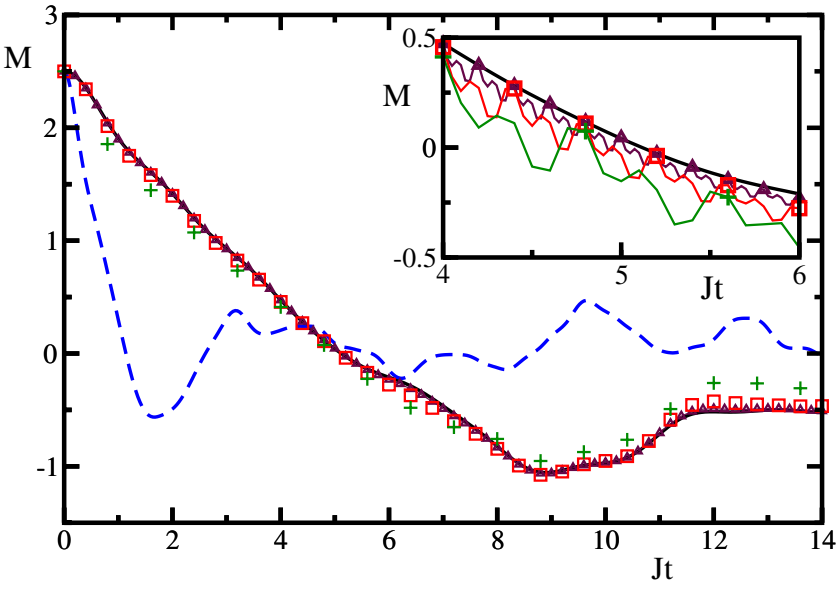

Figure 1. Time evolution of local magnetization for a clean spin-1/2 chain with $\Delta = J/2$ and $L = 10$. Blue dashed line: chaotic system $H_{\rm NN} + H_{\rm NNN}$ in the absence of pulses; black solid line: integrable system $H_{\rm NN}/2$; symbols: chaotic system $H_{\rm NN} + H_{\rm NNN}$ in the presence of pulses (5); triangles: $\tau = 0.025J^{-1}$; squares: $\tau = 0.05J^{-1}$; pluses: $\tau = 0.1J^{-1}$. Main panel: data acquired every $T_n = nT_{\rm c} = 8\tau$; inset: data acquired every $t_n = n\tau$.

The sequence removes NNN couplings, but at the price of also reducing the effects of NN couplings. We achieve the following dominant term for the average Hamiltonian:

$$\bar{H}^{(0)} = \frac{H_{\rm NN}}{2}.$$

In figure 1, we show the evolution of a chaotic chain described by $H_{\rm NN} + H_{\rm NNN}$ under free evolution and subjected to pulses (5), and compare it with the dynamics of an ideal integrable

Table 3. Sign changes of the coupling terms in $H_{\rm NN}$ and $H_{\rm NNN}$ according to pulses (7).

	$(0,\tau)$	$(\tau,2\tau)$	$(2\tau,3\tau)$	$(3\tau,4\tau)$
NN couplings				
$S^x_{k+1}S^x_{k+2}$	+	−	−	+
$S^y_{k+1}S^y_{k+2}$	+	+	−	−
$S^z_{k+1}S^z_{k+2}$	+	−	+	−
NNN couplings				
$S^x_{k+1}S^x_{k+3}$	+	+	+	+
$S^y_{k+1}S^y_{k+3}$	+	+	+	+
$S^z_{k+1}S^z_{k+3}$	+	+	+	+

system given by $H_{\rm NN}/2$. The quantity considered is the local magnetization, which is defined as the magnetization of the first half of the chain,

$$M(t) \equiv \langle\Psi(t)|\sum_{n=1}^{L/2} S^z_n|\Psi(t)\rangle. \tag{6}$$

We choose as initial state $|\Psi(0)\rangle$ a highly excited state far from equilibrium: it consists of spins pointing up in the first half of the chain and pointing down in the other half, so that $M(0) = L/4$. A discussion about control metrics and the dependence of the results on different initial states is given in the appendix.

In order to average out unwanted terms of the Hamiltonian, we need to consider cycle times smaller than the time scale of the free evolution of the system, that is, $JT_{\rm c} < 1$.[2] For the $T_{\rm c}$s considered in figure 1, the agreement between the evolutions of the chaotic system under control pulses and the integrable chain is excellent. As the values of τ increase, higher order terms in the average Hamiltonian become significant, especially at long times, and the pulsed system must eventually separate from the ideal scenario. Even though strategies exist to reduce the effects of higher order terms in $\bar{H}$ [49]–[52], the access to short time intervals between pulses is essential to the success of dynamical decoupling methods.

Note that control sequences are designed to match the aspired evolution at the completion of cycles, but not necessarily in between cycles. This is well illustrated in the inset. The symbols indicate data obtained at every $T_n = nT_{\rm c}$, while the solid lines correspond to the values of $M(t)$ at every $t_n = n\tau$. The largest mismatches between the evolution determined by $H_{\rm NN} + H_{\rm NNN} + H_{\rm c}(t)$ and by $H_{\rm NN}/2$ occur in between cycles.

A second alternative to recover the dynamical behavior of an integrable system consists in canceling out the NN exchange leaving the NNN couplings unaffected. This is achieved with a sequence of four pulses equally separated ($T_{\rm c} = 4\tau$), which alternates rotations around perpendicular directions, say x and y. The pulses can affect only spins in odd sites, or only in even sites, or they alternate between odd and even sites as below,

$$P_1 = P_3 = \prod_{k=0}^{\lfloor (L-1)/2\rfloor} \mathrm{e}^{-\mathrm{i}\pi S^y_{1+2k}}, \quad P_2 = P_4 = \prod_{k=0}^{\lfloor (L-2)/2\rfloor} \mathrm{e}^{-\mathrm{i}\pi S^x_{2+2k}}. \tag{7}$$

Pulses (7) change the signs of the terms of $H_{\rm NN} + H_{\rm NNN}$ as shown in table 3.

[2] The convergence of the expansions used to obtain the evolution operator in the presence of pulse sequences is discussed in [51] and references therein.

The above control sequence leads to two uncoupled chains, one consisting of exchange couplings between spins in the original odd sites and the other consisting of couplings between spins in the original even sites. The first-order term in the average Hamiltonian is simply

$$\bar{H}^{(0)} = H_{\mathrm{NNN}}.$$

4.2. Gapped versus gapless systems

Here, we analyze how the time evolution typical of a system with an energy gap may be achieved from a gapless system subjected to a quantum control scheme. Instead of completely averaging out unwanted terms of the Hamiltonian, as done in the previous section, the goal now is to change the ratio between different coupling strengths. This is accomplished by varying the time interval between pulses, so that the weight of some specific terms may be decreased relatively to others.

4.2.1. Increasing the role of the Ising interaction: $\Delta < 1 \rightarrow \Delta > 1$. Consider a given system in the gapless phase described by H_{NN}, where $\Delta < 1$. To induce the transport behavior of a chain in the gapped phase, we develop a sequence that leaves the Ising interaction unaffected, but reduces the strength of the flip-flop term. The sequence consists of two π-rotations around the z-axis, which affect only spins in even sites (or only in odd sites), that is,

$$P_1 = P_2 = \exp\left(-\mathrm{i}\pi \sum_{k=0}^{\lfloor (L-2)/2 \rfloor} S^z_{2k+2}\right). \tag{8}$$

The first pulse is applied after τ_1 and the second after τ_2, which results in the following reshaped Hamiltonian at $T_{\mathrm{c}} = \tau_1 + \tau_2$:

$$\bar{H}^{(0)} = \sum_{n=1}^{L-1} J \left[\frac{(\tau_1 - \tau_2)}{T_{\mathrm{c}}}\left(S^x_n S^x_{n+1} + S^y_n S^y_{n+1}\right) + \Delta S^z_n S^z_{n+1}\right].$$

The gapped phase is obtained if $T_{\mathrm{c}}\Delta/(\tau_1 - \tau_2) > 1$.

On the left panels of figure 2 we compare the desired evolution of a gapped system described by $H_{\mathrm{NN}}/4$, where $\Delta_{\mathrm{w}} = 2$, with the dynamics of a given gapless system described by H_{NN}, where $\Delta = 1/2$, and subjected to control operations. The pulses, given by equation (8), have time intervals that lead to $T_{\mathrm{c}}\Delta/(\tau_1 - \tau_2) = 2$. The agreement between the two cases at the completion of each cycle is very good for the chosen time intervals, but of course it worsens as T_{c} increases. The price to dynamically achieve a transition from a gapless to a gapped Hamiltonian, however, is the slowing down of the system evolution in the gapped phase by a factor of 4. For a discussion about ballistic versus diffusive transport in XXZ models with small and large values of Δ, see [21, 22] and references therein.

It is also possible to force a transition from a gapped to a gapless phase. For example, pulses (7) separated by different intervals τ_1, $\tau_2 = \tau_4$, and τ_3, reshape the Hamiltonian of a Mott insulator described by H_{NN} with $\Delta > 1$ as follows:

$$\bar{H}^{(0)} = \sum_{n=1}^{L-1} \frac{J}{T_{\mathrm{c}}}\left[(\tau_1 - \tau_3)\left(S^x_n S^x_{n+1} + S^y_n S^y_{n+1}\right) + \Delta(\tau_1 + \tau_3 - 2\tau_2) S^z_n S^z_{n+1}\right].$$

The gapless phase is then obtained if $\Delta(\tau_1 + \tau_3 - 2\tau_2) < (\tau_1 - \tau_3)$. In particular, when $\tau_1 + \tau_3 = 2\tau_2$, we recover the XY model.

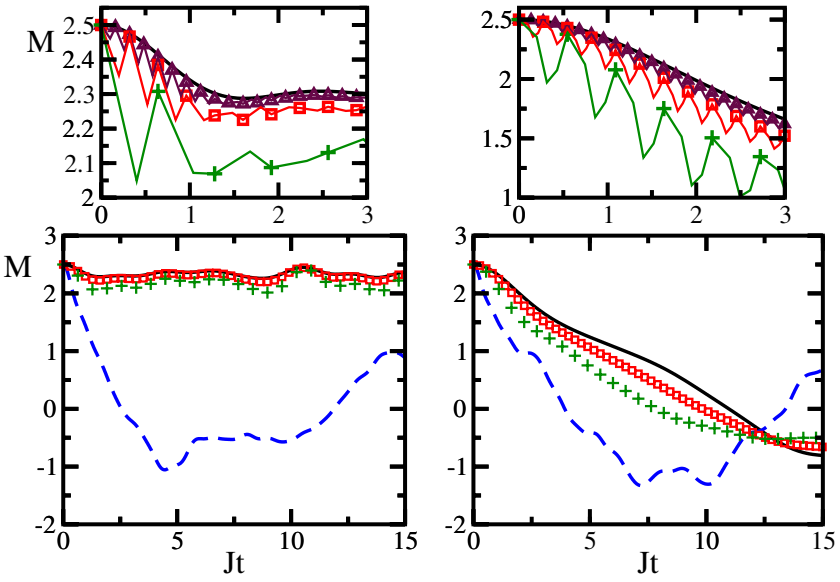

Figure 2. Evolution of local magnetization for a clean spin-1/2 chain with $L = 10$, described by H_{NN} with $\Delta = 1/2$ (left panels) and by $H_{\mathrm{NN}} + \alpha H_{\mathrm{NNN}}$ with $\Delta = 1$ and $\alpha = 0.1$ (right panels). Absence of pulses: blue dashed line; presence of pulses: symbols. Left panels: pulses (8) leading to $T_{\mathrm{c}}\Delta/(\tau_1 - \tau_2) = 2$. Triangles (appearing only in the inset): $\tau_1 = 0.1J^{-1}$ and $\tau_2 = 0.06J^{-1}$; squares: $\tau_1 = 0.2J^{-1}$ and $\tau_2 = 0.12J^{-1}$; pluses: $\tau_1 = 0.4J^{-1}$ and $\tau_2 = 0.24J^{-1}$. Black solid line: system described by $H_{\mathrm{NN}}/4$ with $\Delta_{\mathrm{w}} = 2$. Right panels: pulses (7) leading to $\alpha T_{\mathrm{c}}/(\tau_1 - \tau_{\mathrm{a}}) = 0.64$, where $\tau_{\mathrm{a}} = \tau_{2,3,4} = \tau_1(0.64 - \alpha)/(0.64 + 3\alpha)$. Triangles (appearing only in the inset): $\tau_1 = 0.05J^{-1}$; squares: $\tau_1 = 0.1J^{-1}$; pluses: $\tau_1 = 0.2J^{-1}$. Black solid line: system described by $(H_{\mathrm{NN}} + \alpha_{\mathrm{w}} H_{\mathrm{NNN}})/6.4$ with $\Delta = 1$ and $\alpha_{\mathrm{w}} = 0.64$. Bottom panels: data acquired every cycle; top panels: data acquired every pulse.

4.2.2. From the spin fluid to the dimer phase: $\alpha < \alpha_{\mathrm{c}} \to \alpha > \alpha_{\mathrm{c}}$. Let us now focus on a system described by $H_{\mathrm{NN}} + \alpha H_{\mathrm{NNN}}$ with $\alpha < \alpha_{\mathrm{c}}$ and $\Delta = 1$. Our goal is to apply to it a sequence of control pulses that induces a dynamical behavior similar to that of a system in the dimerized phase ($\alpha > \alpha_{\mathrm{c}}$). A possible sequence consists of the four pulses presented in equation (7), although now separated by different time intervals τ_1 and $\tau_2 = \tau_3 = \tau_4 = \tau_{\mathrm{a}}$. It leads to the following dominant term of the average Hamiltonian at $T_{\mathrm{c}} = \tau_1 + \tau_2 + \tau_3 + \tau_4$,

$$\bar{H}^{(0)} = \frac{(\tau_1 - \tau_{\mathrm{a}})}{T_{\mathrm{c}}} H_{\mathrm{NN}} + \alpha H_{\mathrm{NNN}}.$$

The desired dynamics is reached when $\alpha T_{\mathrm{c}}/(\tau_1 - \tau_{\mathrm{a}}) > \alpha_{\mathrm{c}}$.

On the right panels of figure 2, we compare the dynamics of a system in the gapless phase given by $H_{\mathrm{NN}} + \alpha H_{\mathrm{NNN}}$, where $\alpha = 0.1$ and $\Delta = 1$, with the wanted dynamics of a system in the gapped phase described by $(H_{\mathrm{NN}} + \alpha H_{\mathrm{NNN}})/6.4$, where $\alpha_{\mathrm{w}} = 0.64$ and $\Delta = 1$. Pulses (7) applied to the original system are separated by intervals that satisfy the constraint $\alpha T_{\mathrm{c}}/(\tau_1 - \tau_{\mathrm{a}}) = 0.64$. For $JT_{\mathrm{c}} < 1$, the dynamics of the perturbed gapless system at the completion of the cycles agrees well with the evolution of the gapped system.

Note that our pulsed system will now be in the limit of strong frustration and will therefore become chaotic [46]. We expect an initial state with energy far from the edges of the spectrum to show a dynamical behavior typical of a chaotic system[3], although slowed down by a factor α_w/α.

A variety of new pulse sequences may be designed according to our needs and experimental capabilities. For instance, the sequence described above, which has no effect on NNN couplings, could be combined with a sequence of π-pulses in the z-direction given by

$$P_z = \prod_{k=0}^{\lfloor (L-1)/4 \rfloor} e^{-i\pi S^z_{1+4k}} \prod_{k=0}^{\lfloor (L-2)/4 \rfloor} e^{-i\pi S^z_{2+4k}}.$$

These pulses would affect only the flip-flop term of H_{NNN}. The combined sequence would reduce the strength of H_{NN} and keep only the Ising interaction from H_{NNN}. Hamiltonians similar to this resulting one have been analyzed in the context of transport [53] and thermalization in gapped systems [9].

5. Conclusion

We studied how quantum control methods may be used to manipulate the dynamics of quantum many-body systems. This is accomplished with sequences of unitary transformations designed to achieve a desired evolution operator. The unitary transformations correspond to instantaneous electromagnetic field pulses, which, in the case of spin systems, rotate the spins by a certain angle. The strategy employed to construct the control sequences was the average Hamiltonian theory.

Four target spin-1/2 chains were considered. Two were chaotic, (i) disordered with NN exchange and (ii) clean with NN exchange and NNN frustrating couplings, and two were in the gapless phase, (iii) clean with NN couplings and small Ising interaction and (iv) clean with NN couplings and weak frustration. We discussed sequences of control pulses aiming at averaging out the effects of disorder, in case (i), and the effects of NNN couplings, in case (ii), and then induced in these chaotic systems a dynamical behavior typical of an integrable clean chain with only NN exchange. We also developed new sequences capable of changing the ratio between the coupling strengths of the system. This was done by exploiting, in the same sequence, different time intervals between the pulses, so that the effects of specific terms in the Hamiltonian might be reduced, but not necessarily eliminated. By dynamically decreasing the weight of the flip-flop term in (iii), we obtained a time evolution characteristic of gapped systems with large Ising interaction (also mentioned was a scheme to achieve the dynamics of a metal out of an insulator). By decreasing the strength of the NN couplings with respect to the NNN exchange in (iv), we obtained the dynamical behavior of a chain in the limit of strong frustration (also mentioned was a way to combine to this scheme a sequence that removes the NNN flip-flop term).

Site addressability and the possibility of varying the time intervals between pulses add versatility to dynamical decoupling schemes and open an array of new opportunities for controlling quantum systems. Our focus here was on using these ideas as an additional tool to reach regimes and phases that are relevant in studies of the nonequilibrium dynamics of many-body systems.

[3] For studies about what to expect for the dynamics of frustrated chaotic systems see, for instance [5, 6, 18, 19, 21].

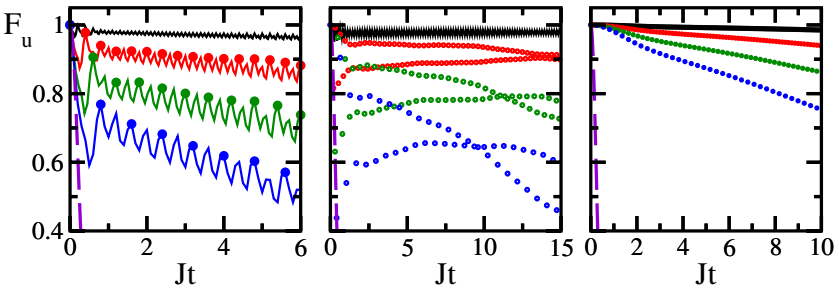

Figure A.1. Propagator fidelity versus time for clean spin-1/2 chains, $L = 10$. Left panel: propagator for $H_w = H_{NN}/2$ versus propagator for a pulsed system described by $H_{NN} + H_{NNN}$; $\Delta = 1/2$ for both. From top to bottom: pulses (5) separated by $\tau = 0.025J^{-1}, 0.05J^{-1}, 0.075J^{-1}$ and $0.1J^{-1}$. Data acquired after every pulse, cycles indicated with symbols. Middle panel: propagator for $H_w = H_{NN}/4$ with $\Delta = 2$ versus propagator for pulsed H_{NN} with $\Delta = 1/2$. Data acquired after every pulse and indicated with symbols. From top to bottom: pulses (8) separated by $\tau_1 = 0.1J^{-1}$ and $\tau_2 = 0.06J^{-1}$; $\tau_1 = 0.2J^{-1}$ and $\tau_2 = 0.12J^{-1}$; $\tau_1 = 0.3J^{-1}$ and $\tau_2 = 0.18J^{-1}$; $\tau_1 = 0.4J^{-1}$ and $\tau_2 = 0.24J^{-1}$. Right panel: propagator for $H_w = (H_{NN} + \alpha_w H_{NNN})/6.4$ with $\alpha_w = 0.64$ versus propagator for pulsed $H_{NN} + \alpha H_{NNN}$ with $\alpha = 0.1$; $\Delta = 1$ for both. From top to bottom: pulses (7) separated by $\tau_1 = 0.025J^{-1}, 0.05J^{-1}, 0.075J^{-1}, 0.1J^{-1}$ and $\tau_a = \tau_{2,3,4} = \tau_1(0.64 - \alpha)/(0.64 + 3\alpha)$. Data acquired after every cycle. All panels: fast decaying dashed line—absence of pulses.

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Appendix. Control metric

In the main text, we showed numerical results for the local magnetization. This choice was motivated by works on transport properties and thermalization, where quantities of experimental interest are usually considered. The decision to take the particular initial state where all up-spins are in the first half of the chain was a way to connect our results with previous transport studies [17]–[19] and also to provide a good illustration, since states where $M(0) \sim 0$ would not give much information. However, in order to quantify how successful the designed control sequences are in general, we need an appropriate control metric that is independent of the initial state.

How close a pulse sequence brings the evolution of a system to a desired dynamics given by $U_w(t)$ may be assessed by the so-called propagator fidelity, which is defined as [54]

$$F_u(t) \equiv |\mathrm{Tr}[U_w^\dagger(t)U(t)]|/D. \tag{A.1}$$

Above, $D = 2^L$ is the dimension of the system and $U(t)$ dictates the evolution of the pulsed system. This quantity deals directly with propagators and therefore avoids any mention to particular initial states. Our goal is to achieve $F_u(t) \to 1$.

In figure A.1, we show again the cases studied in figures 1 and 2, but now from the perspective of the evolution operator. The performance of the pulse sequences deteriorates for

longer T_cs and longer times. For the shortest cycle times considered, however, the agreement between the pulsed and desired dynamics is very good for relatively long times, giving a proof of principle that indeed we may manipulate the dynamics of quantum many-body systems with dynamical decoupling methods[4].

The agreement between $U_w(t)$ and $U(t)$ is best at the completion of cycles, as shown in the left panel and also as verified, but not shown, for the system in the right panel. Interestingly, however, for the system in the middle panel this does not always hold. In the middle panel, we compare the gapless system H_{NN} with $\Delta = 1/2$ and the gapped system $H_w = H_{NN}/4$ with $\Delta = 2$. The applied pulses involve a single direction z and the cycle is completed after only two pulses (cf equation (8)). In the figure, we show with circles the values of F_u after every pulse, so each value of the pair $\tau_{1,2}$ has associated with it two curves. For short times, the performance of the sequence is best at the completion of the cycles, as expected, but at longer times, the scenario is reversed and the best performance appears in between cycles. The origin of this shift of roles, which occurs faster for longer cycle times, deserves further investigation.

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[4] In this work we have not studied how particular initial states may affect our results. Our goal here was to show that control methods are viable techniques for the studies of the dynamics of quantum many-body systems. Studies of best-case performance may be found, for instance, in [55] and references therein.

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