Spectrum, symmetries, and dynamics of Heisenberg spin-1/2 chains

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ArXiv:1209.0115
To appear in the Am. J. Phys.

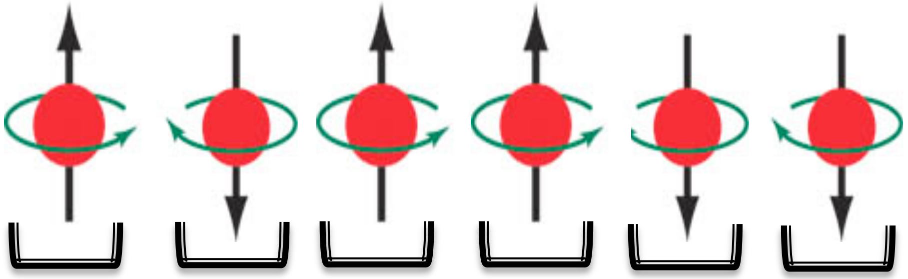
Overview

- Spin-1/2 System - Heisenberg model
- Histograms of spectrum
  - Predict dynamics
- Dynamics

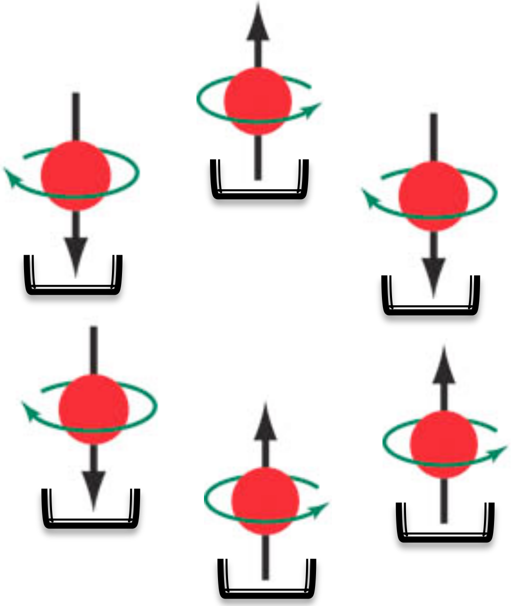
Spin-1/2 chain

$$\uparrow = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \downarrow = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

1D chains of L sites, each site containing one spin



Open chain



Closed chain

Heisenberg model

- Hamiltonian

$$\hat{H} = \sum_{n=1}^{L-1}\Big[\underbrace{J\Delta S_n^z S_{n+1}^z}_{\textbf{Ising interaction}} + \underbrace{J\left(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y\right)}_{\textbf{Flip-flop}}\Big]$$

$S_n^{x,y,z} = \dfrac{\sigma_n^{x,y,z}}{2}$ where $\sigma_n^{x,y,z}$ are the Pauli matrices acting on site *n*

$$\sigma^x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \sigma^y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \quad \sigma^z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

Ising interaction and flip-flop

$$\hat{H} = \sum_{n=1}^{L-1}\left[J\Delta S_n^z S_{n+1}^z + J\left(S_n^x S_{n+1}^x + S_n^y S_{n+1}^y\right)\right]$$

Ising Interaction

$$\sigma^z|\uparrow\rangle \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = +\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\sigma^z|\downarrow\rangle \Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = -\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Flip-flop term

$$\sigma^x|\uparrow\rangle \Rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\sigma^x|\downarrow\rangle \Rightarrow \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\frac{J\Delta}{4}\sigma_n^z\sigma_{n+1}^z|\uparrow\uparrow\rangle = +\frac{J\Delta}{4}|\uparrow\uparrow\rangle$$

$$\frac{J\Delta}{4}\sigma_n^z\sigma_{n+1}^z|\uparrow\downarrow\rangle = -\frac{J\Delta}{4}|\uparrow\downarrow\rangle$$

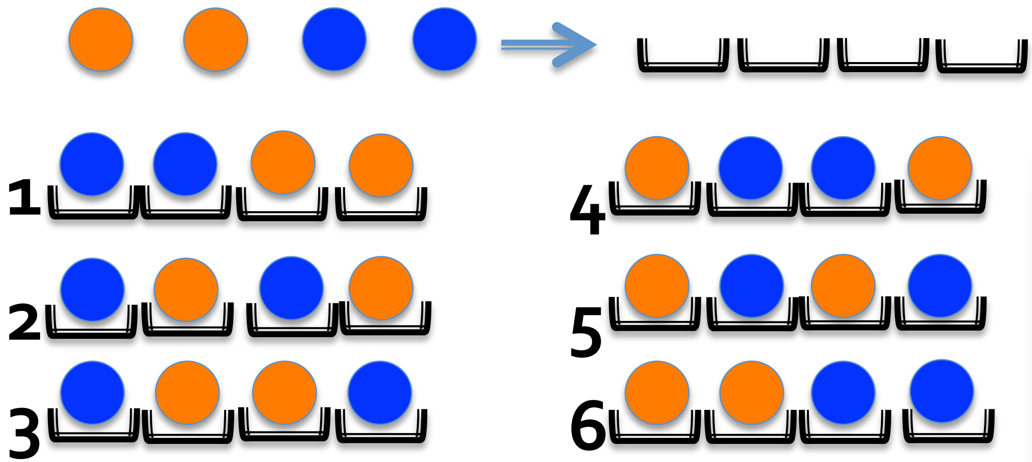
$$\frac{J}{2}\sigma_n^x\sigma_{n+1}^x|\uparrow\downarrow\rangle \longrightarrow \frac{J}{2}|\downarrow\uparrow\rangle$$

Hamiltonian in form of a matrix

(closed chain)

- Basis (L=4, N=2)
  - Total number of distinct configurations

$$\begin{pmatrix} L \\ N \end{pmatrix} = \frac{L!}{(N!)(L-N)!}$$



$$\begin{pmatrix} 0 & J/2 & 0 & 0 & J/2 & 0 \\ J/2 & -J\Delta & J/2 & J/2 & 0 & J/2 \\ 0 & J/2 & 0 & 0 & J/2 & 0 \\ 0 & J/2 & 0 & 0 & J/2 & 0 \\ J/2 & 0 & J/2 & J/2 & -J\Delta & J/2 \\ 0 & J/2 & 0 & 0 & J/2 & 0 \end{pmatrix}$$

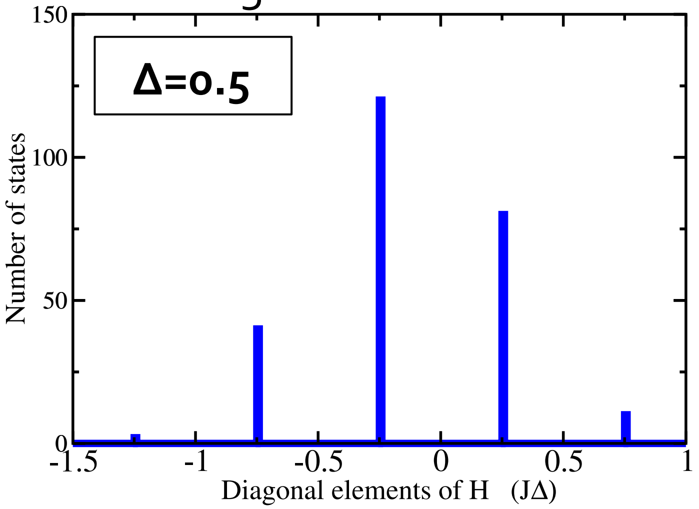
$$H_{11} = \langle \uparrow \uparrow \downarrow \downarrow | H | \uparrow \uparrow \downarrow \downarrow \rangle = 0$$

$$H_{12} = \langle \uparrow \uparrow \downarrow \downarrow | H | \uparrow \downarrow \uparrow \downarrow \rangle = \frac{J}{2}$$

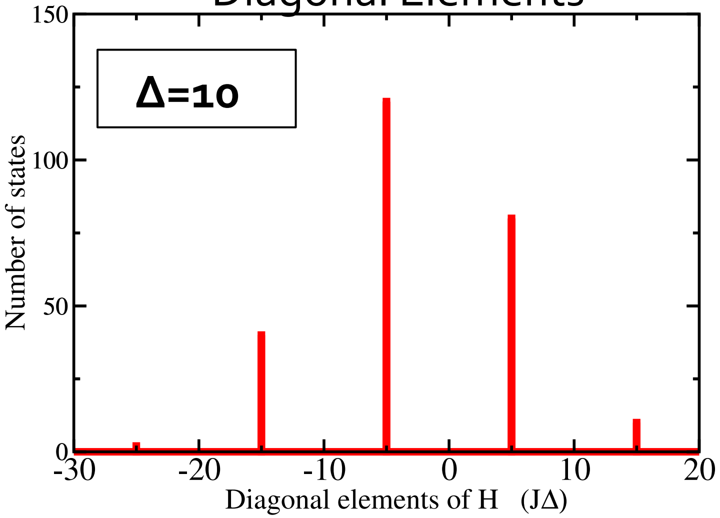
$$H_{22} = \langle \uparrow \downarrow \uparrow \downarrow | H | \uparrow \downarrow \uparrow \downarrow \rangle = -J\Delta$$

Spectrum – predicting the dynamics J=1, L=10, N=5, closed chain

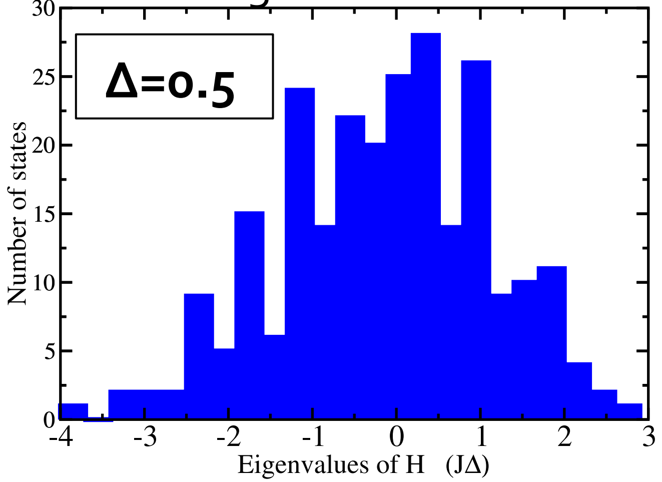
Diagonal Elements



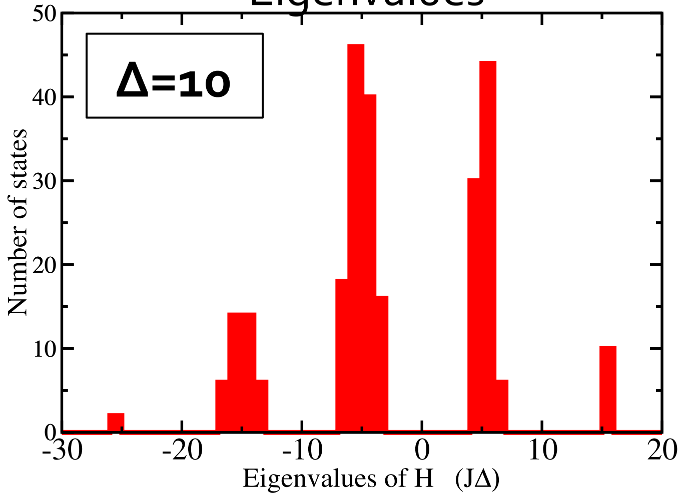
Diagonal Elements



Eigenvalues

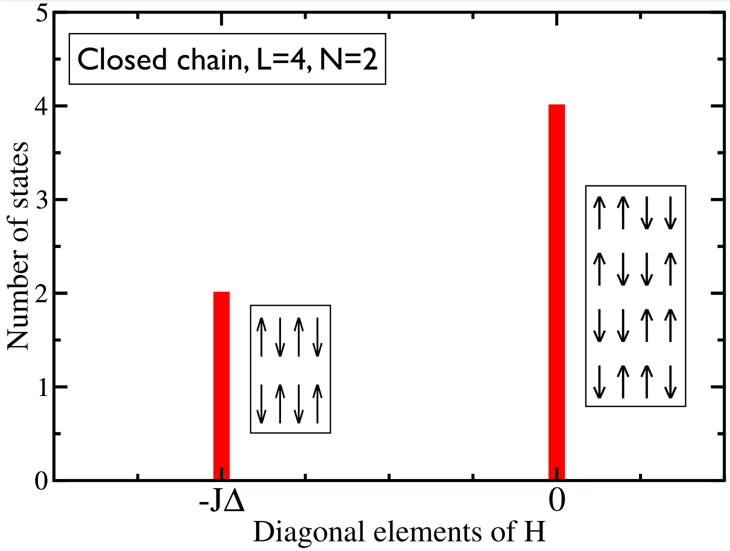


Eigenvalues



Dynamics

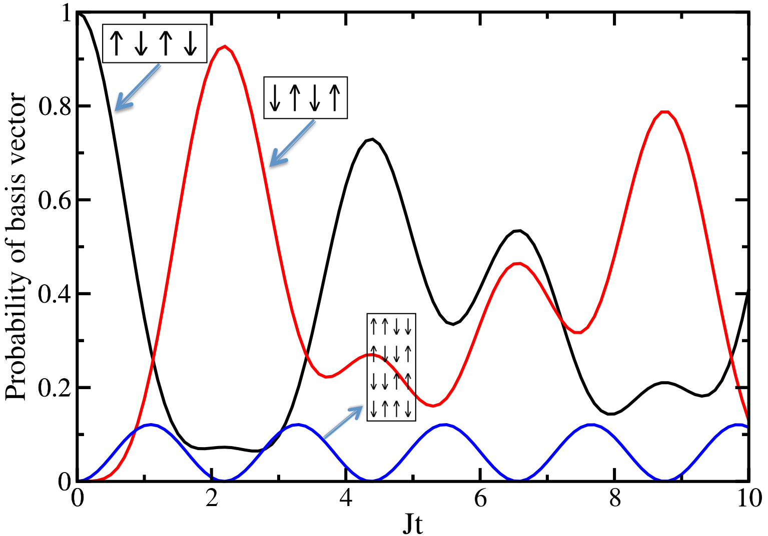
(Closed chain, L=4, N=2)



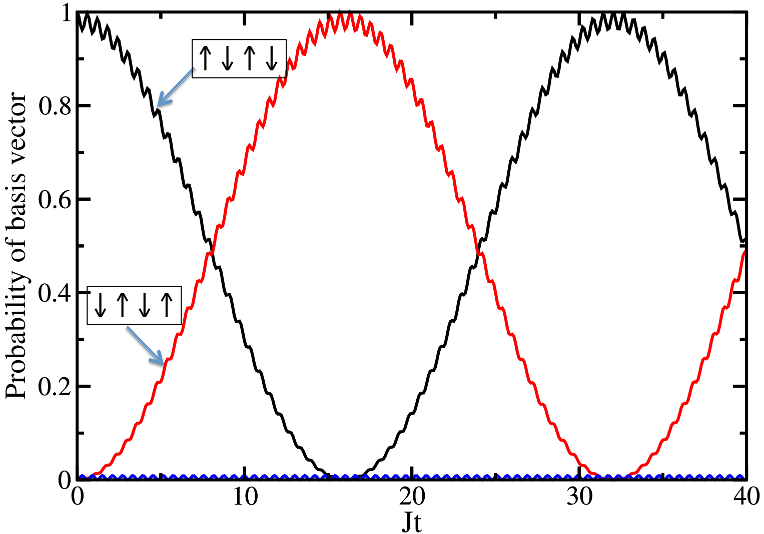
Initial state $\Psi(0)$

$$i\hbar\frac{\partial}{\partial t}\Psi(r,t)=\left[\frac{-\hbar^2}{2m}\nabla^2+V(r,t)\right]\Psi(r,t)$$

Δ=0.5



Δ=10



Symmetries

- Total spin in z-direction

$$\mathrm{S}_z = \sum_{n=1}^{L} S_z^n$$

$$[H, \mathrm{S}_z] = 0$$

- Parity
- Global Rotation

Additional Research

ArXiv:1209.0115

- Other observables like magnetization
- Different system sizes
- Border effects

Acknowledgements

- Henry Kressel Research Scholarship for funding this project